Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Financial Economics 68 (2003) 161–199
JOURNAL OF Financial ECONOMICS
www.elsevier.com/locate/econbase

Style investing ☆

Nicholas Barberis[a,*], Andrei Shleifer[b]

[a] Graduate School of Business, University of Chicago, Chicago, IL 60637, USA
[b] Department of Economics, Harvard University, Cambridge, MA 02138, USA

Received 28 November 2000; received in revised form 11 December 2001

Abstract

We study asset prices in an economy where some investors categorize risky assets into different styles and move funds among these styles depending on their relative performance. In our economy, assets in the same style comove too much, assets in different styles comove too little, and reclassifying an asset into a new style raises its correlation with that style. We also predict that style returns exhibit a rich pattern of own- and cross-autocorrelations and that while asset-level momentum and value strategies are profitable, their style-level counterparts are even more so. We use the model to shed light on several style-related empirical anomalies.

JEL classification: G11, G12, G14

Keywords: Style investing; Comovement; Positive feedback; Value; Momentum

1. Introduction

One of the clearest mechanisms of human thought is classification, the grouping of objects into categories based on some similarity among them (Rosch and Lloyd, 1978; Wilson and Keil, 1999). We group countries into democracies and dictatorships based on features of political systems within each group. We classify

☆ We have benefited from the comments of two anonymous referees and from discussions with John Campbell, Doug Diamond, Eugene Fama, Edward Glaeser, Will Goetzmann, Sanford Grossman, Rafael La Porta, David Laibson, Sendhil Mullainathan, Geert Rouwenhorst, Lawrence Summers, Jeffrey Wurgler, and seminar participants at the University of Chicago, Harvard University, London Business School, Princeton University, the University of Iowa, Wharton, the AFA, and the NBER.

*Corresponding author. Tel.: +1-773-834-0677; fax: +1-773-702-0458.
E-mail address: nick.barberis@gsb.uchicago.edu (N. Barberis).

0304-405X/03/$ - see front matter
doi:10.1016/S0304-405X(03)00064-3

occupations as blue collar or white collar based on whether people work primarily with their hands or with their heads. We put foods into categories such as proteins and carbohydrates based on their nutritional characteristics.

Classification of large numbers of objects into categories is also pervasive in financial markets. When making portfolio allocation decisions, many investors first categorize assets into broad classes such as large-cap stocks, value stocks, government bonds, and venture capital and then decide how to allocate their funds across these various asset classes (Bernstein, 1995; Swensen, 2000). The asset classes that investors use in this process are sometimes called ''styles,'' and the process itself, namely allocating funds among styles rather than among individual securities, is known as ''style investing.'' In this paper, we analyze financial markets in which many investors pursue style investing.

Assets in a style or class typically share a common characteristic, which can be based in law (e.g., government bonds), in markets (e.g., small-cap stocks), or in fundamentals (e.g., real estate). In some cases, the cash flows of assets in the same style are highly correlated, as with automotive industry stocks, while in other cases, such as closed-end funds, they are largely uncorrelated. Some styles are relatively permanent over the years (e.g., U.S. government bonds), while others come (e.g., small stocks) and go (e.g., railroad bonds). One reason for the appearance of a new style is financial innovation, as when mortgage-backed securities were invented. Another reason is the detection of superior performance in a group of securities with a common characteristic: small stocks became a more prominent investment style following Banz' (1979) discovery of the small firm effect. Styles typically disappear after long periods of poor performance, as was the case with railroad bonds.

There are at least two reasons why both institutional and individual investors might pursue style investing. First, categorization simplifies problems of choice and allows us to process vast amounts of information reasonably efficiently (Mullainathan, 2000). Allocating money across ten asset styles is far less intimidating than choosing among the thousands of listed securities. Second, the creation of asset categories helps investors evaluate the performance of professional money managers, since a style automatically creates a peer group of managers who pursue that particular style (Sharpe, 1992). Money managers are now increasingly evaluated relative to a performance benchmark specific to their style, such as a growth or a value index.

These benefits of style investing are particularly attractive to institutional investors, such as pension plan sponsors, foundations, and endowments, who as fiduciaries must follow systematic rules of portfolio allocation. Perhaps for this reason, interest in style investing has grown over the years, paralleling the growth of institutional investors. Not surprisingly, the financial services industry has responded to the interest. Most pension fund managers, as well as some mutual fund managers catering to the needs of individual investors, now identify themselves as following a particular investment style, such as growth, value, or technology.[1]

[1] As indicated earlier, we use the term ''style investor'' to refer to investors such as pension plan sponsors who allocate funds at the style level, rather than at the individual asset level. The term can also be used in a

The growing importance of style investing points to the usefulness of assessing its effect on financial markets and security valuation. This paper presents a simple model that allows for such an assessment. The model combines style-based portfolio selection strategies of investors with a plausible mechanism for how these investors choose among styles. Specifically, we assume that many investors allocate funds based on *relative* past performance, moving into styles that have performed well in the past, and financing this shift by withdrawing funds from styles that have performed poorly. We also assume that these fund flows affect prices.

These simple assumptions generate a number of empirical predictions, some already available in the theoretical literature, others entirely new. In our model, style investing generates common factors in the returns of assets that happen to be grouped into the same style. These return factors can be completely unrelated to common factors in cash flows (they exist even if there is *no* common component to underlying cash flows) and can be accompanied by higher *average* returns for reasons that have nothing to do with risk. When an asset is reclassified into a new style, it comoves more with that style after reclassification than before, even if the cash-flow covariance matrix is unchanged. And while style investing increases the correlation between assets in the same style, it lowers the correlation between assets in different styles.

We also predict a rich structure of style return autocorrelations: positive own-autocorrelations and negative cross-autocorrelations in the short run, and with the opposite signs in the long run. The predictions about own-autocorrelations are shared with earlier models, while those about cross-autocorrelations are more unique to our framework. Moreover, while asset-level momentum and value strategies are profitable in our model, as in other models, we make the additional prediction that style-level momentum and value strategies can be as profitable or even more profitable than their asset-level counterparts.

Our predictions about time-series autocorrelations reflect the fact that in our economy, investment styles follow a specific life cycle. The birth of a style is often triggered by good fundamental news about the securities in the style. The style then matures as its good performance recruits new funds, further raising the prices of securities belonging to the style. Finally, the style collapses, either because of arbitrage or because of bad fundamental news. Over time, the style can be reborn.

We use our model to shed light on a number of puzzling empirical facts. Among other phenomena, we address the common factors in small stock and value stock returns that appear unrelated to common factors in cash flows (Fama and French, 1995), the performance of the small stock investment style over time, the poor returns of value stocks in 1998 and 1999 despite good earnings (Chan et al., 2000), and patterns of comovement when stocks are added to indices such as the S&P 500.

(*footnote continued*)
related, but distinct sense to describe money managers who restrict themselves to picking stocks from within a specific asset style. While both uses of the term are common in practice, in this paper "style investor" refers only to the investors providing the funds and not to the money managers they hire.

Of the two assumptions underlying our predictions—investors' policy of allocating funds at the style level and their doing so based on relative past performance—neither has received much prior attention in the theoretical literature. The closest papers to our own are De Long et al. (1990a) and Hong and Stein (1999), in which investors allocate across assets based on *absolute* past performance. Neither of these papers studies the effect of classifying assets into styles, nor the effect of relative rather than absolute performance-chasing.[2]

In Section 2, we construct a simple model of style investing. Section 3 develops some of the intuition that lies behind the model's predictions. In Section 4, we lay out the model's implications in a series of formal propositions. Section 5 analyzes two specific kinds of styles—indices and price-dependent styles—in more detail. Section 6 concludes.

2. A model of style investing

2.1. Assets and styles

We consider an economy with $2n$ risky assets in fixed supply and a riskless asset, cash, in perfectly elastic supply and with zero net return. Following Hong and Stein (1999), we model risky asset i as a claim to a single liquidating dividend $D_{i,T}$ to be paid at some later time T. The eventual dividend equals

$$D_{i,T} = D_{i,0} + \varepsilon_{i,1} + \cdots + \varepsilon_{i,T}, \tag{1}$$

where $D_{i,0}$ and $\varepsilon_{i,t}$ are announced at time 0 and time t, respectively, and where

$$\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{2n,t})' \sim \mathrm{N}(0, \Sigma_D), \quad \text{i.i.d. over time.} \tag{2}$$

The price of a share of risky asset i at time t is $P_{i,t}$ and the return on the asset between time $t-1$ and time t is[3]

$$\Delta P_{i,t} = P_{i,t} - P_{i,t-1}. \tag{3}$$

We assume that, to simplify their decision-making, some investors in the economy group the risky assets into a small number of categories, which we refer to as styles, and express their demand for risky assets at the level of these styles. In other words, a style is a group of risky assets that some investors do not distinguish between when formulating their demand.

To test any predictions that emerge from a model of style investing, it is important to have a concrete way of identifying styles. One way of doing this is to look at the products that mutual and pension fund managers offer clients. If money managers are responsive to their clients, they will create products that correspond to the categories those clients like to use. The fact that many money managers offer funds

[2] Empirical work on styles has advanced more rapidly than theoretical work on the topic. Recent contributions to the empirical literature include Brown and Goetzmann (1997, 2001) and Chan et al. (2002).

[3] For simplicity, we refer to the asset's change in price as its return.

that invest in small-cap stocks suggests that "small stocks" is a style in the minds of many investors. Large stocks, value stocks, growth stocks, and stocks within a particular industry, country, or index are then also all examples of styles.

We build a simple model of style investing. There are two styles, X and Y, and each risky asset in the economy belongs to one, and only one, of these two styles. Risky assets 1 through n are in style X while $n+1$ through $2n$ are in style Y. For now, we assume that this classification is permanent, so that the composition of the two styles is the same in every time period. It may be helpful to think of X and Y as "old economy" stocks and "new economy" stocks, say.[4]

As a measure of the value of style X at time t, we use $P_{X,t}$, the average price of a share across all assets in style X,

$$P_{X,t} = \frac{1}{n}\sum_{l \in X} P_{l,t}. \tag{4}$$

The return on style X between time $t-1$ and time t is

$$\Delta P_{X,t} = P_{X,t} - P_{X,t-1}. \tag{5}$$

Although our model does not require it, we restrict attention to simple cash-flow covariance structures. In particular, we suppose that the cash-flow shock to an asset has three components: a market-wide cash-flow factor which affects assets in both styles, a style-specific cash-flow factor which affects assets in one style but not the other, and an idiosyncratic cash-flow shock specific to a single asset. Formally, for $i \in X$,

$$\varepsilon_{i,t} = \psi_M f_{M,t} + \psi_S f_{X,t} + \sqrt{(1-\psi_M^2-\psi_S^2)} f_{i,t}, \tag{6}$$

and for $j \in Y$,

$$\varepsilon_{j,t} = \psi_M f_{M,t} + \psi_S f_{Y,t} + \sqrt{(1-\psi_M^2-\psi_S^2)} f_{j,t}, \tag{7}$$

where $f_{M,t}$ is the market-wide factor, $f_{X,t}$ and $f_{Y,t}$ are the style-specific factors, and $f_{i,t}$ and $f_{j,t}$ are idiosyncratic shocks. The constants ψ_M and ψ_S control the relative importance of the three components. Each factor has unit variance and is orthogonal to the other factors, so that

$$\sum\nolimits_D^{ij} \equiv \operatorname{cov}(\varepsilon_{i,t}, \varepsilon_{j,t}) = \begin{cases} 1, \ i=j, \\ \psi_M^2 + \psi_S^2, \ \ i, j \text{ in the same style}, \ i \neq j, \\ \psi_M^2, \ \ i, j \text{ in different styles}. \end{cases} \tag{8}$$

In words, all assets have a cash-flow news variance of one, the pairwise cash-flow correlation between any two distinct assets in the same style is the same, and the pairwise cash-flow correlation between any two assets in different styles is also the same.

[4] More generally, a given security may belong to multiple overlapping styles. A small bank stock with a low price-earnings ratio may be part of a small stock style, a financial industry style, and a value style. A model capturing such overlaps can be constructed and would yield similar but less transparent predictions.

The results we derive later do not require that styles be associated with cash-flow factors. However, if the purpose of styles is to simplify decision-making, it is plausible that investors might create them by grouping together assets with similar cash flows.

2.2. Switchers

There are two kinds of investors in our model, "switchers" and "fundamental traders." The investment policy of switchers has two distinctive characteristics. First, they allocate funds at the level of a style. Second, how much they allocate to each style depends on that style's past performance *relative to other styles*. In other words, each period, switchers allocate more funds to styles with better than average performance and finance these additional investments by taking funds away from styles with below average performance. To capture this, we write their demand for shares of asset i in style X at time t as

$$N^S_{i,t} = \frac{1}{n}\left[A_X + \sum_{k=1}^{t-1} \theta^{k-1}\left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}\right)\right] = \frac{N^S_{X,t}}{n}, \quad (9)$$

where A_X and θ are constants with $0<\theta<1$. Symmetrically, switcher demand for shares of asset j in style Y at time t is

$$N^S_{j,t} = \frac{1}{n}\left[A_Y + \sum_{k=1}^{t-1} \theta^{k-1}\left(\frac{\Delta P_{Y,t-k} - \Delta P_{X,t-k}}{2}\right)\right] = \frac{N^S_{Y,t}}{n}. \quad (10)$$

In words, when deciding on their time t allocation, switchers compare style X's and style Y's return between time $t-2$ and time $t-1$, between time $t-3$ and time $t-2$, and so on, with the most recent past being given the most weight. They then move funds into the style with the better prior record, buying an equal number of shares of each asset in that style and reducing their holdings of the other style. The fact that their demand for all assets within a style is the same underscores our assumption that they allocate funds at the style level and do not distinguish among assets in the same style. The parameter θ determines how far back they look when comparing the past performance of styles and hence, the persistence of their flows. A_X and A_Y can be thought of as their average long-run demand for styles X and Y, respectively, from which they deviate based on the styles' relative performance.[5]

We think of the relative performance feature in Eqs. (9) and (10) as arising from extrapolative expectations, whereby switchers think that future style returns will be similar to past style returns, combined with switchers' reluctance to let their allocations to the broadest asset classes – cash, bonds, and stocks – deviate from preset target levels. Put differently, this second condition means that while switchers are quite willing to move between different equity styles, they are much less willing to

[5] The strategies in Eqs. (9) and (10) are not self-financing. Rather, we assume that switchers are endowed with sufficient resources to fund their strategies. This allows us to abstract from issues which are not our main focus here – the long-run survival of switchers, for example – and to concentrate on understanding the behavior of prices when switchers do play a role in setting them.

change their *overall* allocation to equities. Institutional investors in particular try to keep their allocations to the three broadest asset classes close to predetermined targets (Swensen, 2000).

The intuition for how extrapolative expectations and target allocation levels combine to give the allocations in Eqs. (9) and (10) is straightforward. Holding everything else constant, an increase in $\Delta P_{X,t-1}$, the most recent past return for old economy stocks, leads switchers to forecast higher returns on that style in the future and hence to increase their demand for it at time t. However, since they want to keep their overall allocation to equities unchanged, they have to sell shares of new economy stocks, style Y. Therefore, $\Delta P_{X,t-1}$ has an opposite effect on $N^S_{i,t}$ in Eq. (9) and $N^S_{j,t}$ in Eq. (10), making demand a function of relative past performance.[6]

Extrapolative expectations can themselves be motivated by a cognitive bias that leads investors to put more weight on past returns than they should when forecasting future returns. For example, people often estimate the probability that a data set is generated by a certain model by the degree to which the data is *representative* or reflects the essential characteristics of the model (Tversky and Kahneman, 1974). A style which has had several periods of high returns is representative of a style with a high true mean return, which may explain why impressive past returns raise some investors' forecasts of future returns.

The same behavior can also stem from agency considerations. An institutional investor, such as the sponsor of a defined benefit plan, may move into styles with good past performance and out of styles with poor performance simply became such strategies are easier to justify ex-post to those monitoring their actions.

Although there is still relatively little work analyzing data on institutional fund flows, the available research supports the idea that investors move funds towards styles with strong past performance. Choe et al. (1999) and Froot et al. (2001), for example, show that foreign institutional investors tend to buy into countries with good recent stock market performance.

In reality, investors have many styles to choose from, not just two. Even with many styles, though, the two-style formulation in Eqs. (9) and (10) remains relevant. When an investor pours money into a style he deems attractive, he may finance this by withdrawing funds from just *one* other style, rather than from many others. One reason for this is transaction costs. In terms of withdrawal fees and time spent, it is likely to be less costly to take \$10 million away from one money manager than to take \$1 million away from ten of them.

Another, potentially more important, reason is that there is often a natural candidate style to withdraw funds from, namely a style's *twin style*. Many styles come in natural pairs. Stocks with a high value of some characteristic constitute one style, and stocks with a low value of the same characteristic, the twin. Value stocks and growth stocks are a simple example. When an investor moves into the growth style, the value style is a tempting source of funds. First, because of the way twins are

[6] The appendix in Barberis and Shleifer (2000) shows more formally how extrapolative expectations combined with a constraint on asset class allocations leads to demand functions like those in Eqs. (9) and (10).

defined, there is no overlap between them. Second, it is easy to succumb to the mistaken belief that since a style and its twin are defined as opposites, their returns will also be "opposite": if prospects for the growth style are good, prospects for the value style must be bad.

2.3. Fundamental traders

The second investor type in our model is *fundamental traders*. They act as arbitrageurs and try to prevent the price of each asset from deviating too far from its expected final dividend.

We assume that, at the start of each period, fundamental traders have CARA preferences defined over the value of their invested funds one period later. Our justification for giving them a short horizon is drawn from Shleifer and Vishny (1997), who argue that if investors are not sophisticated enough to understand a money manager's strategies, they will use short-term returns as a way of judging his competence and withdraw funds after poor performance. The threat of this happening forces arbitrageurs to take a short-term view.

Fundamental traders therefore solve

$$\max_{N_t} E_t^{\mathrm{F}}(-\exp[-\gamma(W_t + N_t'(P_{t+1} - P_t))]), \tag{11}$$

where

$$N_t = (N_{1,t}, \ldots, N_{2n,t})', \tag{12}$$

$$P_t = (P_{1,t}, \ldots, P_{2n,t})', \tag{13}$$

and where $N_{i,t}$ is the number of shares allocated to risky asset i, γ governs the degree of risk aversion, E_t^{F} denotes fundamental trader expectations at time t, and W_t is time t wealth.

If fundamental traders assume a Normal distribution for conditional price changes, optimal holdings N_t^{F} are given by

$$N_t^{\mathrm{F}} = \frac{(V_t^{\mathrm{F}})^{-1}}{\gamma}(E_t^{\mathrm{F}}(P_{t+1}) - P_t), \tag{14}$$

where

$$V_t^{\mathrm{F}} = \mathrm{var}_t^{\mathrm{F}}(P_{t+1} - P_t), \tag{15}$$

with the F superscript in V_t^{F} again denoting a forecast made by fundamental traders.

2.4. Prices

Given fundamental trader expectations about future prices, which we discuss shortly, prices are set as follows. The fundamental traders serve as market makers and treat the demand from switchers as a supply shock. If the total supply of the $2n$ assets is given by the vector Q, Eq. (14) implies

$$P_t = E_t^{\mathrm{F}}(P_{t+1}) - \gamma V_t^{\mathrm{F}}(Q - N_t^S), \tag{16}$$

where $N_t^S = (N_{1,t}^S, \ldots, N_{2n,t}^S)'$. In contrast to switchers, who form expectations of future prices based on past prices, fundamental traders are forward looking and base price forecasts on expectations about the final dividend. One way they may do this is to roll Eq. (16) forward iteratively, setting

$$E_{T-1}^{\mathrm{F}}(P_T) = E_{T-1}^{\mathrm{F}}(D_T) = D_{T-1}, \tag{17}$$

where $D_t = (D_{1,t}, \ldots, D_{2n,t})'$. This leads to

$$P_t = D_t - \gamma V_t^{\mathrm{F}}(Q - N_t^S) - E_t^{\mathrm{F}} \sum_{k=1}^{T-t-1} \gamma V_{t+k}^{\mathrm{F}}(Q - N_{t+k}^S). \tag{18}$$

Suppose that fundamental traders set

$$V_\tau^{\mathrm{F}} = V, \ \forall \tau, \tag{19}$$

where V has the same structure as the cash-flow covariance matrix Σ_D, so that V^{ij}, its (i, j)th element, is given by

$$V^{ij} = \begin{cases} \sigma^2, \ i = j \\ \sigma^2 \rho_1, \ i, \ j \text{ in the same style, } i \neq j \\ \sigma^2 \rho_2, \ i, \ j \text{ in different styles} \end{cases} \tag{20}$$

and also that they set

$$E_t^{\mathrm{F}}\left(N_{t+k}^S\right) = \bar{N}^S. \tag{21}$$

Eq. (21) means that while fundamental traders recognize the existence of a supply shock due to switchers, they are not sophisticated enough to figure out its time series properties. They simply lean against the shock, preventing it from pushing prices too far away from expected cash flows.

Our assumptions imply

$$P_t = D_t - \gamma V(Q - N_t^S) - (T - t - 1)\gamma V(Q - \bar{N}^S). \tag{22}$$

Dropping the non-stochastic terms, we obtain

$$P_t = D_t + \gamma V N_t^S. \tag{23}$$

For the particular form of V conjectured by fundamental traders, this simplifies further. Up to a constant, the price of an asset i in style X is

$$\begin{aligned} P_{i,t} &= D_{i,t} + \gamma\sigma^2(1 - \rho_1 + n(\rho_1 - \rho_2)) \frac{N_{X,t}^S}{n} \\ &= D_{i,t} + \frac{1}{\phi} \sum_{k=1}^{t-1} \theta^{k-1} \left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2} \right), \end{aligned} \tag{24}$$

where

$$\phi = \frac{n}{\gamma\sigma^2(1 - \rho_1 + n(\rho_1 - \rho_2))}, \tag{25}$$

and the price of an asset j in style Y is

$$P_{j,t} = D_{j,t} + \frac{1}{\phi}\sum_{k=1}^{t-1} \theta^{k-1}\left(\frac{\Delta P_{Y,t-k} - \Delta P_{X,t-k}}{2}\right). \tag{26}$$

We study equilibria in which fundamental traders' choices of V and $\bar{N}^S$ in Eqs. (19) and (21) are reasonable, in that they lead, through Eq. (22), to prices for which the conditional covariance matrix of returns actually is V, and for which unconditional mean switcher demand actually is $\bar{N}^S$. Such equilibria exist for a wide range of values of the exogeneous parameters Σ_D, A_X, A_Y, θ, and γ.

In a world with only fundamental traders,

$$P_t = D_t. \tag{27}$$

We refer to this as the fundamental value of the assets and denote it P_t^*.

Eq. (23) shows that fundamental traders are unable to push prices back to fundamental values. Their short one-period horizon forces them to worry about shifts in switcher sentiment and makes them less aggressive in combating mispricing, a mechanism originally suggested by De Long et al. (1990b). Their inability to wipe out the influence of noise traders is consistent with the substantial body of empirical evidence indicating that uninformed demand shocks influence security prices (Harris and Gurel, 1986; Shleifer, 1986; Kaul et al., 2000; Lamont and Thaler, 2003). Moreover, if we think of switchers as institutions chasing the best-performing style, our model is consistent with evidence that demand shifts by institutions in particular influence security prices (Gompers and Metrick, 2001).

Even if we included more sophisticated arbitrageurs in our model – arbitrageurs who understand the form of the demand function in Eq. (9) – they might exacerbate rather than counteract the mispricing. This is the finding of De Long et al. (1990a), who consider an economy with positive feedback traders similar to our switchers, as well as arbitrageurs. When an asset's price rises above fundamental value, the arbitrageurs do not sell or short the asset. Rather, they *buy* it, knowing that the price rise will attract more feedback traders next period, leading to still higher prices, at which point the arbitrageurs can exit a profit. Since sophisticated arbitrageurs may amplify rather than counteract the effect of switchers, we exclude them from our simple model.

2.5. Parameter values

In section 4, we prove some general propositions about the behavior of asset prices in our economy. To illustrate some of these propositions, we use a numerical implementation of Eqs. (24) and (26) in which the exogeneous parameters Σ_D, A_X, A_Y, θ, and γ are assigned specific values.

From Eq. (8), the cash-flow covariance matrix is completely determined by ψ_M and ψ_S. We set $\psi_M = 0.25$ and $\psi_S = 0.5$, which gives

$$\sum_D = \begin{pmatrix} \Sigma_0 & \Sigma_1 \\ \Sigma_1 & \Sigma_0 \end{pmatrix}, \tag{28}$$

where

$$\sum\nolimits_0 = \begin{pmatrix} 1 & 0.31 & \cdots & 0.31 \\ 0.31 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0.31 \\ 0.31 & \cdots & 0.31 & 1 \end{pmatrix}, \quad \sum\nolimits_1 = \begin{pmatrix} 0.06 & \cdots & \cdots & 0.06 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0.06 & \cdots & \cdots & 0.06 \end{pmatrix}. \tag{29}$$

The remaining parameters are set equal to

$$A_X = A_Y = 0, \tag{30}$$

$$\theta = 0.95, \tag{31}$$

$$\gamma = 0.093. \tag{32}$$

Eq. (9) shows that θ controls the persistence of switcher flows; a θ close to 1 indicates a high level of persistence. Fundamental trader risk aversion γ is set so that in equilibrium, returns exhibit a level of excess volatility that is reasonable given historical U.S. data. For these parameter values, style returns have a standard deviation 1.3 times the standard deviation of cash-flow shocks.[7] In this equilibrium, the value of ϕ in Eq. (25) is 1.25.[8]

3. Competition among styles

3.1. Impulse response functions

As a first step towards understanding the effect of switchers on asset prices, we use the formulae for price in Eqs. (24) and (26) to generate some impulse response functions. We take $n = 50$, so that there are 100 risky assets, the first 50 of which are in style X and the last 50 in style Y. X and Y can again be thought of as old economy and new economy stocks, respectively. The parameters are set equal to the values in Eqs. (28–32). Fig. 1 shows how the prices $P_{X,t}$ and $P_{Y,t}$ of styles X and Y, defined in Eq. (4), evolve after a one-time cash-flow shock to style X when $t = 1$.

[7] For comparison, the standard deviation of aggregate dividend growth from 1926 to 1995 is close to 12% while the standard deviation of aggregate stock returns over the same period is close to 18%, 1.5 times higher.

[8] The parameter values in Eqs. (28–32) also support other equilibria, including one where returns are only slightly more volatile than cash flows. The intuition is that if fundamental traders think that returns are not very volatile, they will trade against switchers more aggressively, with the result that equilibrium returns will indeed have low volatility. To support the equilibrium described in the main text, we need fundamental traders to expect returns to be substantially more volatile than cash flows. Nevertheless, the results in this paper remain valid across multiple equlibria.

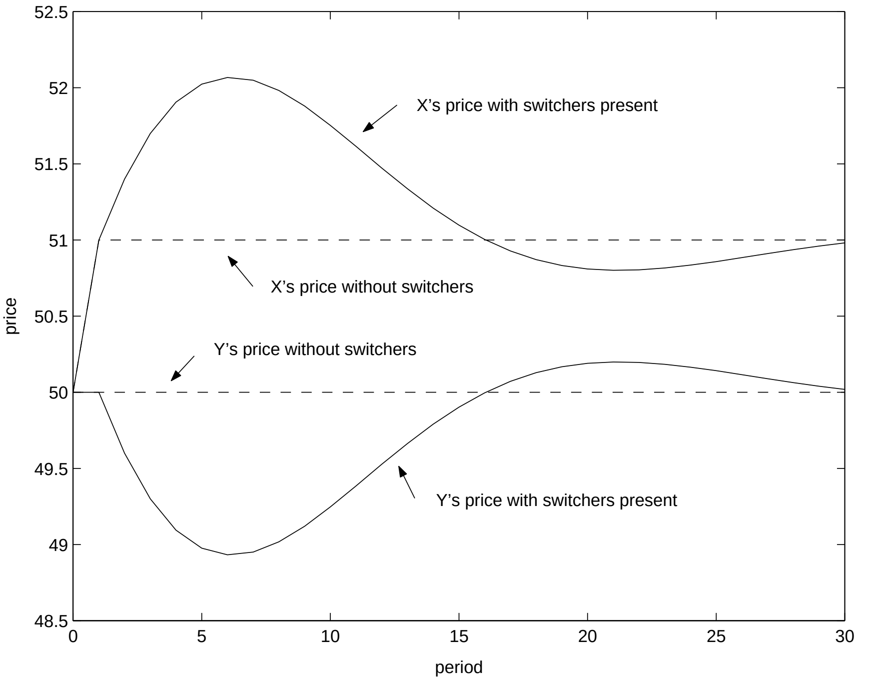

Fig. 1. Impulse responses to a style-level cash-flow shock. Some investors in the economy, known as switchers, group risky assets into two categories, X and Y, known as styles. The solid lines show how the prices of styles X and Y evolve after a cash-flow shock to style X in period 1. Prices of both X and Y are initially 50. For comparison, the dashed lines indicate fundamental values, or prices in the absence of switchers.

In the notation of our model,

$$\varepsilon_{i,1} = 1,\ \varepsilon_{i,t} = 0,\ t > 1,\ \forall i \in X, \tag{33}$$

$$\varepsilon_{j,t} = 0,\ \forall t, \forall j \in Y. \tag{34}$$

We assume $D_{i,0} = 50,\ \forall i$.

The solid line in the top half of the graph tracks $P_{X,t}$, the value of style X in the presence of switchers. The dashed line in the top half is the fundamental value of style X, $P^*_{X,t}$, defined through Eqs. (27) and (4) as the value of style X when there are only fundamental traders in the economy and no switchers.

The figure shows that in the presence of switchers, a cash-flow shock to style X leads to a substantial and long-lived deviation of X's price from its fundamental value. The good cash-flow news about X pushes up its price. This outperformance catches the attention of switchers, who then increase their demand for X in the following period, pushing X's price still higher and drawing in more switchers. In the absence of any more good cash-flow news, switchers' interest in style X eventually fades and prices return to fundamental value.

The fact that switchers' investment decisions are based on *relative* rather than absolute past performance leads to a more novel prediction which we refer to as an *externality*. Fig. 1 shows that the cash-flow shock to X affects not only X's price, but also Y's, even though there has been no news about Y. The good news about X draws funds into that style. However, since switchers want to maintain a constant overall allocation to equities, they finance the extra investment in X by taking money out of Y. This pushes Y's price down, making it look even worse relative to X and leading to still more redemptions by switchers.

In practice, the quantitative magnitude of the externality depends heavily on how investors finance their style shifts. If they finance a shift into a particular style by withdrawing small amounts of money from all other styles, the externality is dispersed and therefore hard to detect. However, if as we argue in Section 2.2, investors finance shifts into a style by withdrawing funds from the style's twin alone, the externality is concentrated and more easily detectable.[9]

We can also look at impulse responses to *asset-specific* cash-flow news. Suppose that asset 1, a member of style X, experiences a one-time cash-flow shock at time 1. In our notation,

$$\varepsilon_{i,1} = 1,\ \varepsilon_{i,t} = 0,\ t > 1, \quad \text{for } i = 1, \tag{35}$$

$$\varepsilon_{i,t} = 0,\ \forall t, \quad \text{for } i = 2, \ldots, 2n. \tag{36}$$

Fig. 2 plots prices $P_{i,t}$ and fundamental values $P^*_{i,t}$ for $i = 1, 2$, and 100. Recall that assets 1 and 2 are in style X while asset 100 is in style Y.

Fig. 2 helps bring out the differences between our model and the related positive feedback trading models of De Long et al. (1990a) and Hong and Stein (1999), in which the feedback occurs at the level of an individual asset, so that noise traders increase their demand for an asset if it had a good return in the previous period. In these earlier models, a cash-flow shock to asset 1 only pushes asset 1's price away from fundamental value. Asset 1's outperformance attracts the attention of positive feedback traders who then buy the asset in the next period, pushing its price up too high. Assets 2 and 100, on the other hand, are unaffected.

Fig. 2 shows that in our model, *all three* assets deviate from fundamental value after the initial cash-flow shock to asset 1. The fact that assets 2 and 100, which received no cash-flow news at all, also move away from fundamental value is due to the two new features of our model: a demand function that is defined at the style level and a focus on relative, rather than absolute, performance. The time 1 cash-flow shock to asset 1 boosts not only asset 1's return but also style X's return, attracting attention from switchers, who then allocate more funds to style X at time 2, pushing both assets 1 and 2 away from fundamental value. Since the inflows to X are financed by withdrawing funds from Y, the price of asset 100 is pushed below fundamental value.

[9] Another kind of externality arises when a sector experiences a positive shock to investment opportunities, drawing in capital and driving up interest rates, which then pushes risky asset prices in other sectors down. Our model makes the distinct prediction that the externality will be concentrated in a style's twin and not be dispersed across all other risky assets.

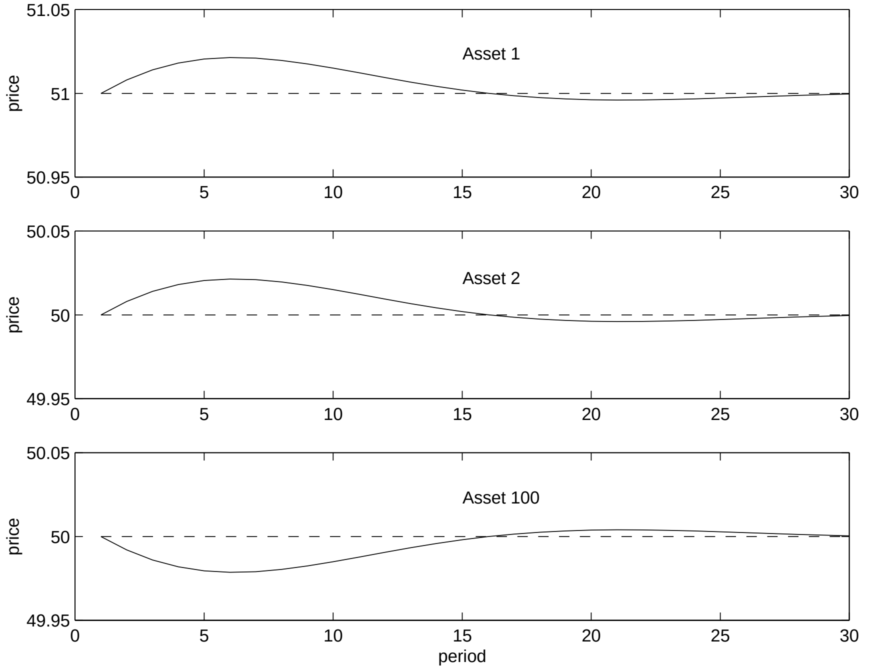

Fig. 2. Impulse responses to an asset-level cash-flow shock. Some investors in the economy, known as switchers, group risky assets into two categories, X and Y, known as styles. The solid lines show how the prices of assets 1, 2 and 100 evolve after a cash-flow shock to asset 1 in period 1. Assets 1 and 2 are in style X while asset 100 is in style Y. All assets have an initial price of 50 at time 0. For comparison, the dashed lines indicate fundamental values, or prices in the absence of switchers.

3.2. Discussion

The impulse response functions show that, in our model, styles go through cycles. A style X is set in motion by good fundamental news about itself or alternatively by bad news about another style Y, which affects it through the externality. The style then swings away from fundamental value for a prolonged period, powered by fund flows attracted by its superior past performance. Finally, the style returns to fundamental value because of selling by fundamental traders, because of bad news about its own fundamentals, or most interestingly, because of good news in a competing style Y, which draws attention and investment dollars away from X.

In some cases, the cycles we describe may be reinforced by academic work analyzing the historical performance of a style. It is noteworthy that Banz' (1979) study on the outperformance of small-cap stocks was followed by several years of strikingly good returns on that style. Our model explains this by saying that Banz' study attracted the attention of switchers, who diverted funds to small stocks, pushing them higher, thus drawing in yet more switchers and generating a long period of superior performance. Of course, our model also predicts that these good returns should eventually be reversed, and it is interesting that after 1983, small-cap stocks experienced two decades of lackluster returns. Indeed, the poor returns of small-cap stocks after 1983 are crucial for distinguishing the style investing story from other explanations of the 1975 to 1983 outperformance, for example that investors were simply pushing small stocks up to their correct values after learning that they had been underpriced for many years.

More radically, cycles may be set in motion by data snooping, as analysts looking through historical data identify abnormal returns. When analysts succeed in convincing investors that they have found strategies earning true superior returns, they will recruit new resources to the strategies, thereby confirming the anomaly for a period of time. Perhaps the discovery of the size effect in the 1970s is an example of such creation of a style out of what might have been a fluke in the data.

The externality from style switching is helpful in interpreting other recent evidence. During 1998 and 1999, value stocks performed extremely poorly by historical standards, lagging both growth stocks and the broad index by a significant margin. As Chan et al. (2000) show, this poor performance occurred despite the fact that the earnings growth and sales growth of value stocks over this period were as high as those of growth stocks and unusually good by historical standards. In other words, the poor performance of value portfolios cannot be easily linked to their fundamentals. A more natural explanation comes from our theory. The poor performance of value stocks in 1998 and 1999 could have been due to the spectacular performance of large growth stocks which generated large flows of funds—unrelated to fundamentals—into these stocks and out of value, the obvious competing style.

Another example comes, once again, from the historical performance of small stocks. Siegel (1999) argues that one reason for the vastly superior performance of small stocks relative to large stocks during 1975 to 1983 was the dismal performance of the "Nifty Fifty" large-cap growth stocks in 1973 and 1974. The demise of these high profile large stocks left investors disenchanted with the large stock style and generated a flow of funds into the competing style, small stocks, triggering a small stock cycle.[10] A competing increase in the relative demand for large stocks, prompted by the rise of indexation and institutional investing more generally, may have arrested this wave of high small stock returns. According to Gompers and Metrick (2001), institutional investors prefer large stocks and their ownership of these stocks has increased rapidly in the last 20 years. This increase in demand for the competing style could be one reason for the poor relative performance of small stocks after 1983.

[10] In fact, the large-cap growth style did not perform significantly worse than other styles during 1973 and 1974. Siegel's argument depends on investors mistakenly perceiving the large stock style as a poor performer, perhaps because of the Nifty Fifty's very high visibility.

4. The behavior of asset prices

We now present a systematic analysis of the effect of switcher flows on asset prices. We summarize the predictions of our model in a series of propositions, focusing on predictions that are largely unique to our framework.

To illustrate the propositions, we use simulated data. As in Section 3, we take $n = 50$, so that there are 100 risky assets, the first 50 of which are in style X and the last 50 in style Y. For the parameter values in Eqs. (28–32), we use the price formulae in Eqs. (24) and (26) to simulate long time series of prices for the 100 assets.

4.1. Comovement within styles

Since switcher demand for securities is expressed at the level of a style, the prices of assets that are in the same style comove more than the assets' cash flows do. If style X has had superior past performance, switchers invest more in *all* securities in style X, pushing their prices up together. The coordinated demand generates comovement over and above that induced by cash-flow news. More formally, we can prove[11]

Proposition 1. *If two assets i and j, $i \neq j$, belong to the same style, then*

$$\mathrm{corr}(\Delta P_{i,t} - \Delta P_{M,t},\ \Delta P_{j,t} - \Delta P_{M,t}) > \mathrm{corr}(\varepsilon_{i,t} - \varepsilon_{M,t},\ \varepsilon_{j,t} - \varepsilon_{M,t}), \tag{37}$$

where

$$\Delta P_{M,t} = \frac{1}{2n}\sum_{l=1}^{2n} \Delta P_{l,t},\ \ \varepsilon_{M,t} = \frac{1}{2n}\sum_{l=1}^{2n} \varepsilon_{l,t}. \tag{38}$$

In our simulated data, the correlation matrix of market-adjusted returns is

$$\mathrm{corr}([\Delta P_{1,t} - \Delta P_{M,t}, \ldots, \Delta P_{2n,t} - \Delta P_{M,t}]) = \begin{pmatrix} R_0 & R_1 \\ R_1 & R_0 \end{pmatrix}, \tag{39}$$

where

$$R_0 = \begin{pmatrix} 1 & 0.34 & \cdots & 0.34 \\ 0.34 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0.34 \\ 0.34 & \cdots & 0.34 & 1 \end{pmatrix}, \quad R_1 = \begin{pmatrix} -0.35 & \cdots & \cdots & -0.35 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ -0.35 & \cdots & \cdots & -0.35 \end{pmatrix}. \tag{40}$$

In the presence of switchers, the correlation between the market-adjusted returns of distinct assets in the *same* style is 0.34. This is indeed higher than the assets' market-adjusted cash-flow news correlation of 0.14.

The proposition is stated in terms of market-adjusted returns and cash-flow news because in general, two assets in the same style may comove more than their cash flows simply because of a market-level return factor induced by changes in interest

[11] Proofs of all propositions are in the appendix.

rates or risk aversion. Controlling for market-level effects allows us to focus on style-level factors, although the proposition also holds for raw returns and cash-flow news.

Proposition 1 suggests a novel way of understanding common factors in the returns of groups of assets. Such common factors are usually interpreted as reflecting common factors in the assets' cash-flow news. In our framework, they arise simply because the assets in question form a natural style and are therefore subject to style-level switcher flows that make them comove more than their cash flows. In particular, Proposition 1 implies that there can be a common factor in the returns of a group of assets even if the assets' cash-flow news are completely uncorrelated.

If there *is* a common factor in the cash flows of assets within a style, not only will the common factor in returns be more pronounced, as per Proposition 1, but it need not be strongly related to the common factor in cash flows. Since this period's style return is in part driven by switcher flows and hence by *past* returns, it need not line up with this period's cash-flow news.

The idea that style investing generates comovement in returns unrelated to comovement in cash flows has significant implications for the interpretation of security returns. Fama and French (1993) find a striking common factor in the returns of small stocks as well as a clear common component in value stock returns. The simplest rational pricing view of this comovement holds that it must be due to common factors in the underlying earnings of small stocks and value stocks. Fama and French (1995) test this explanation and obtain surprising results. Although they do find common factors in the cash-flow news of small stocks and value stocks, they uncover little evidence that the return factors are driven by the cash-flow factors.

These results do not sit well with the view that return comovement is driven by cash-flow comovement, but emerge very naturally from a style investing perspective. This view holds that, to the extent that small firms belong to a style (because size is a characteristic defining a style), they move together by virtue of fund flows in and out of that style. Even if there is a common component in the earnings of small firms, it need not be related to the common component in their returns. This is exactly Fama and French's (1995) finding.

A common factor in the returns of value stocks unrelated to a common factor in their earnings can arise in two distinct ways in our framework. The more obvious mechanism relies on ‘‘value stocks’’ being a style in itself. An alternative story holds that industries are the most important styles and therefore that *they* have common factors in returns only weakly related to cash flows. Since value stocks belong to industries which have fallen out of favor among switchers, they exhibit a common factor in returns unrelated to cash flows simply by inheriting that property from industry styles. Either mechanism explains Fama and French's results for value stocks.[12]

[12] In principle, rational changes in discount rates can also generate comovement. However, changes in interest rates or risk aversion induce a common factor in the returns to *all* stocks and do not explain why a particular group of stocks comoves. A common factor in news about the risk of the assets in a style may be a source of comovement for those assets, but there is little direct evidence to support such a mechanism in the case of small or value stocks.

Other evidence is also consistent with this analysis. Pindyck and Rotemberg (1990) find comovement in the prices of different commodities over and above what can be explained by economic fundamentals. Lee et al. (1991) find that the prices of closed-end funds move together even when the net asset values of the funds are only weakly correlated. In the language of the present model, if investors classify all commodities into a "commodity" style and all closed-end funds into a "closed-end fund" style, and then move money in and out of these styles, the coordinated demand induces a common component in returns even when the assets' fundamentals have little in common.

Also relevant are the findings of Froot and Dabora (1999), who study "twin" stocks such as Royal Dutch and Shell. These stocks are claims to the same cash-flow stream, but are primarily traded in different locations. Royal Dutch is traded most heavily in the U.S. and Shell in the U.K. In a frictionless market, these stocks should move together. Froot and Dabora show, however, that Royal Dutch is more sensitive to movements in the U.S. market while Shell comoves more with the U.K. market. A style-based perspective provides a natural explanation. Royal Dutch, a member of the S&P 500, is buffeted by the flows of investors for whom the S&P 500 is a style and therefore comoves more with this index. For the same reason, Shell, a member of the FTSE index, comoves more with that index.[13]

Proposition 1 is driven by our assumption that investors classify assets into styles and then allocate funds at the style level. Traditional models of positive feedback trading in which the feedback occurs at the individual asset level cannot easily deliver Proposition 1. In these models, asset returns are typically only as correlated as the underlying cash flows.

Another important class of models assumes that investors are uncertain about the growth rate of an asset's cash flows and are forced to learn it by observing cash-flow realizations. After several periods of impressive cash-flow growth, for example, investors raise their estimate of the rate (Veronesi, 1999; Brennan and Xia, 2001; Lewellen and Shanken, 2002). If learning occurs at the level of individual assets, these models also have trouble delivering Proposition 1, as asset returns are again only as correlated as cash flows.

In models that combine learning with bounded processing ability (Peng and Xiong, 2002), investors simplify the allocation problem by creating categories of assets with correlated cash flows and then learning about cash flows at the category level. In other words, there is information pooling (Shiller, 1989). Since investors allocate some of their funds by category, such models can explain why a group of stocks with a strong common factor in cash flows might have an even stronger common factor in returns. However, they are hard-pressed to explain why there

[13] Another prediction of our model is that keeping the cash-flow covariance matrix constant, an increase in the importance of style investing should increase the fraction of a stock's volatility that is due to common, rather than idiosyncratic, shocks. Campbell et al. (2001) find that over the past three decades, firm-specific volatility has risen relative to total volatility. They argue that this is most likely due to the fact that the cash-flow covariance matrix *has* changed, with firm-specific cash-flow news becoming more volatile.

would be a common factor in the returns of assets whose cash flows are largely uncorrelated, such as closed-end funds, since investors would be unlikely to create a category out of such assets in the first place.

Some existing models *can* explain why assets with uncorrelated cash flows might move together. Kyle and Xiong (2001) propose a theory of comovement based on the idea that financial intermediaries experience wealth effects. When intermediaries suffer trading losses, their risk-bearing capacity is reduced, leading them to sell assets across the board and inducing correlation in fundamentally unrelated securities. This model seems appropriate for understanding why apparently unrelated assets trading in different countries comove strongly in times of financial crisis, such as August 1998. It is less plausible an explanation for why small stocks move together, regardless of economic conditions.

Finally, in their study of closed-end funds, Lee et al. (1991) propose another view of comovement. Their view is related to our own, in that they assume that shifts in uninformed demand affect prices, but it is nevertheless distinct. They argue that some groups of securities may only be held by a particular subset of all investors, such as individual investors. As these investors' risk aversion or sentiment changes, they change their exposure to the risky assets that they hold, inducing a common factor in the returns to these securities. In other words, this theory predicts that there will be a common factor in the returns of securities that are held primarily by a specific class of investors. This is distinct from our own theory, which predicts a common factor in the returns of securities that many investors classify as a style, even if these securities are in *all* investors' portfolios.

Lee et al.'s (1991) theory is well-suited to explaining why small stocks and closed-end funds comove, as both of these asset classes are held almost entirely by individual investors. Indeed, style investing is a less plausible explanation, since small stocks and closed-end funds do not form a natural single style. On the other hand, style investing may be a better way of thinking about the common factor in value stocks, since there is no evidence that these securities are held primarily by a particular investor class.

The style investing view of comovement has other predictions and implications for the interpretation of empirical facts. Not only should stocks within a style comove more than their cash flows do, but stocks that *enter* a style should comove more with the style after they are added to it than before. For example, a stock that is added to an index such as the S&P 500 should comove more with the index after it is added than before. Changes in comovement after a security is added to a style provide one of the more unique empirical predictions of our theory. More formally, we can prove:

Proposition 2. *Suppose that asset j, not previously a member of style X, is reclassified as belonging to X. Then* $\mathrm{cov}(\Delta P_{j,t}, \Delta P_{X,t})$ *increases after j is added to style X.*

In our analysis so far, we have taken assets 1 through n to be in style X and assets $n+1$ through $2n$ to be in style Y. In our simulated data for this economy, we find that for any asset j not in style X, in other words, for $j = n+1, \ldots, 2n$,

$$\text{cov}(\Delta P_{j,t},\ \Delta P_{X,t}) = -0.17. \tag{41}$$

Suppose that asset 1 is reclassified into style Y and that asset $n+1$ is reclassified into style X. We now recompute asset $n+1$'s covariance with style X. More specifically, we keep the cash-flow covariance matrix fixed, and use Eqs. (24) and (26) to simulate prices as before, the only difference being the new composition of styles X and Y. We find that[14]

$$\text{cov}(\Delta P_{n+1,t},\ \Delta P_{X,t}) = 0.30, \tag{42}$$

showing that stock $n+1$'s covariance with style X does indeed increase after it is added to that style.

Proposition 2 may also help us differentiate the two views of value stock comovement suggested earlier. If it arises because "value stocks" is itself a style, the proposition predicts that stocks that become value stocks will comove more after entering that category than before. If it arises because industries are styles, there will be no increased comovement after an industry enters the value category. Daniel and Titman's (1997) evidence is more supportive of the latter view. They find that stocks in the value category today comove roughly as much as they did five years earlier.

4.2. Comovement across styles

Two assets in the *same* style, then, will be more correlated than their underlying cash flows. The opposite is true of two assets in *different* styles, asset i in style X, say, and asset j in style Y. Such assets will be *less* correlated than their underlying cash flows. The reason for this is the externality generated by switchers. A good return for style X leads to a flow out of Y and into X, driving the styles in opposite directions and lowering the correlation between them. More formally, we can prove:

Proposition 3. *If assets i and j are in different styles, then*

$$\text{corr}(\Delta P_{i,t} - \Delta P_{M,t},\ \Delta P_{j,t} - \Delta P_{M,t}) < \text{corr}(\varepsilon_{i,t} - \varepsilon_{M,t},\ \varepsilon_{j,t} - \varepsilon_{M,t}). \tag{43}$$

The prediction is stated in terms of market-adjusted returns, not raw returns. It is tempting to think that the externality makes returns on small stocks and large stocks and returns on value stocks and growth stocks pairwise less correlated than their cash flows. However, reality may be more complicated because of overlap between styles. Competition between value and growth suggests that their returns should be less correlated than their fundamentals, but value stocks and growth stocks are both part of the overall U.S. stock market, itself a style. By Proposition 1, this would tend to make value and growth stocks *more* correlated than their cash flows. In view of this complication, we make our prediction in terms of market-adjusted returns. In other words, we predict that the market-adjusted returns on value and growth stocks

[14] There will be a spurious increase in $\text{cov}(\Delta P_{n+1,t},\ \Delta P_{X,t})$ arising from the fact that after reclassification, $\Delta P_{n+1,t}$ enters into the computation of $\Delta P_{X,t}$. The change from -0.17 to 0.30, however, is far in excess of the mechanical effect.

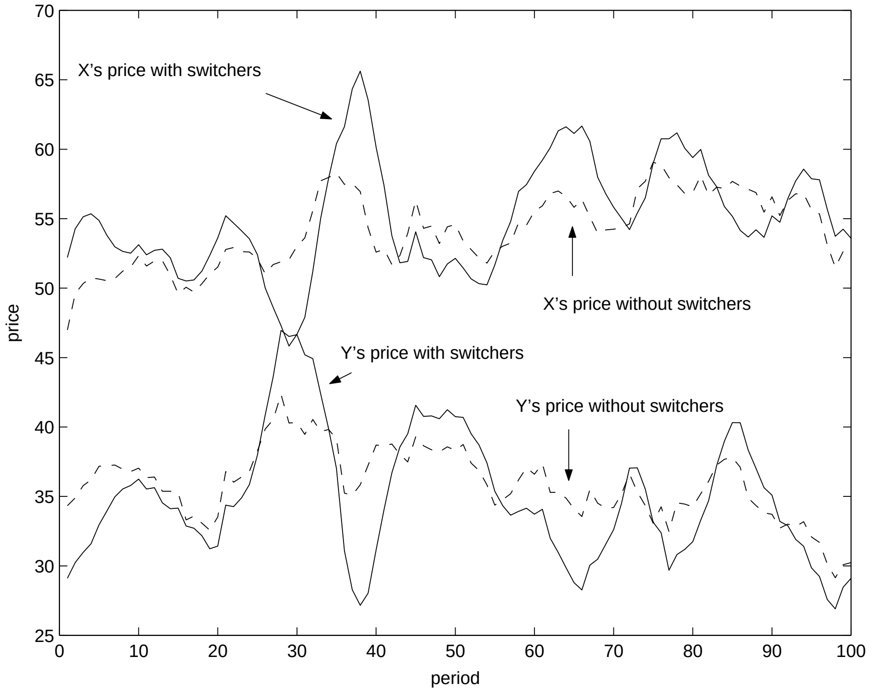

Fig. 3. Price paths for two styles, X and Y. Some investors in the economy, known as switchers, group risky assets into two categories, X and Y, known as styles. The solid lines show the prices of styles X and Y over a 100-period segment of simulated data. For comparison, the dashed lines indicate fundamental values, or prices in the absence of switchers.

are less correlated than the cash flows of value and growth stocks, in turn adjusted for market cash flows.

Eq. (39) shows that for any assets i in X and j in Y,

$$\text{corr}(\Delta P_{i,t} - \Delta P_{M,t}, \; \Delta P_{j,t} - \Delta P_{M,t}) = -0.35, \tag{44}$$

while in our simulated data,

$$\text{corr}(\varepsilon_{i,t} - \varepsilon_{M,t}, \; \varepsilon_{j,t} - \varepsilon_{M,t}) = -0.16, \tag{45}$$

confirming that market-adjusted returns are indeed less correlated than market-adjusted cash flows. Fig. 3, which plots the prices of styles X and Y over a one-hundred-period segment of the simulated data, provides another view of the same phenomenon. The price paths of styles X and Y tend to move in opposite directions.

4.3. Own- and cross-autocorrelations

The presence of switchers in the economy makes style returns positively autocorrelated in the short run and negatively autocorrelated in the long run. A good return for style X draws in switchers who push its price up again next period, inducing positive autocorrelation. The price swing is eventually reversed, generating long-run mean-reversion.

Our model's predictions about own-autocorrelations are not unique to our framework. They can also arise in traditional positive feedback trading models or conventional learning models so long as the assets in a style share a common cash-flow factor. However, the relative performance rule driving switcher flows, and the externality that it creates, lead to more unique predictions about *cross*-autocorrelations across styles, namely that they should be negative in the short run and positive in the long run. A good return on style X at time t generates outflows from Y into X, pushing Y's price down at time $t+1$. In the long run, Y's price recovers, generating positive cross-autocorrelations at longer lags. In summary, we can show:

Proposition 4. *For some* $K \geqslant 1$,

$$\begin{aligned} &\text{(i)} \quad \mathrm{corr}(\Delta P_{X,t},\ \Delta P_{X,t-k}) > 0,\ 1 \leqslant k \leqslant K, \\ &\qquad \mathrm{corr}(\Delta P_{X,t},\ \Delta P_{X,t-k}) < 0,\ k = K+1, \\ \text{and } &\text{(ii)} \quad \mathrm{corr}(\Delta P_{X,t},\ \Delta P_{Y,t-k}) < 0,\ 1 \leqslant k \leqslant K, \\ &\qquad \mathrm{corr}(\Delta P_{X,t},\ \Delta P_{Y,t-k}) > 0,\ k = K+1. \end{aligned} \tag{46}$$

Table 1 shows the magnitude of these own- and cross-autocorrelations for our particular numerical example. The first-order own-autocorrelation is 0.5, while the

Table 1
Own- and cross-autocorrelations
Some investors in the economy, known as switchers, group risky assets into two categories, X and Y, known as styles. For a particular set of model parameter values, the table reports the own- and cross-autocorrelations of raw and market-adjusted returns on styles X and Y.

	$\mathrm{corr}(\Delta P_{X,t}, \Delta P_{X,t-k})$	$\mathrm{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k})$
$k=1$	0.50	−0.50
$k=2$	0.36	−0.36
$k=3$	0.23	−0.23
$k=4$	0.12	−0.12
$k=5$	0.02	−0.02
$k=6$	−0.06	0.06
$k=7$	−0.11	0.11
$k=8$	−0.15	0.15
$k=9$	−0.18	0.18
	$\mathrm{corr}(\Delta P_{X,t}-\Delta P_{M,t}, \Delta P_{X,t-k}-\Delta P_{M,t-k})$	$\mathrm{corr}(\Delta P_{X,t}-\Delta P_{M,t}, \Delta P_{Y,t-k}-\Delta P_{M,t-k})$
$k=1$	0.78	−0.78
$k=2$	0.56	−0.56
$k=3$	0.36	−0.36
$k=4$	0.18	−0.18
$k=5$	0.02	−0.02
$k=6$	−0.10	0.10
$k=7$	−0.19	0.19
$k=8$	−0.24	0.24
$k=9$	−0.28	0.28

correlation of returns nine lags apart is –0.18. It is easy to show that $\text{corr}(\Delta P_{X,t}, \Delta P_{X,t-k}) = -\text{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k})$ for $k \geqslant 1$, a pattern that is clearly visible in the table.

The available evidence on own-autocorrelations is generally supportive of part (i) of Proposition 4. Lo and MacKinlay (1988) and Poterba and Summers (1988) find U.S. monthly stocks returns to be positively autocorrelated at the first lag and negatively autocorrelated thereafter. Cutler et al. (1991) show that monthly returns of international stock and bond indices, as well as of real estate and commodity markets are positively autocorrelated at lags of up to a year, and negatively autocorrelated thereafter. Finally, Lewellen (2002) finds that monthly returns on industry and size-sorted portfolios are positively autocorrelated at a one month lag and negatively autocorrelated after that.

Testing part (ii) is again complicated by style overlaps: in general, stocks will be affected both by flows into the U.S. stock market as a whole, as well as by intra stock market flows between styles. A more robust version of Proposition 4 would therefore be in terms of market adjusted returns:

Proposition 5. *For some* $K \geqslant 1$,

$$\begin{aligned} &\text{(i)} \quad \text{corr}(\Delta P_{X,t} - \Delta P_{M,t},\ \Delta P_{X,t-k} - \Delta P_{M,t-k}) > 0, \quad 1 \leqslant k \leqslant K, \\ &\qquad \text{corr}(\Delta P_{X,t} - \Delta P_{M,t},\ \Delta P_{X,t-k} - \Delta P_{M,t-k}) < 0, \quad k = K+1, \\ \text{and } &\text{(ii)} \quad \text{corr}(\Delta P_{X,t} - \Delta P_{M,t},\ \Delta P_{Y,t-k} - \Delta P_{M,t-k}) > 0, \quad 1 \leqslant k \leqslant K, \\ &\qquad \text{corr}(\Delta P_{X,t} - \Delta P_{M,t},\ \Delta P_{Y,t-k} - \Delta P_{M,t-k}) < 0, \quad k = K+1. \end{aligned} \tag{47}$$

Table 1 presents the magnitudes of own- and cross-autocorrelations of market-adjusted returns in simulated data.

4.4. *Asset-level momentum and value strategies*

We now analyze the profitability of momentum and value strategies that are implemented at the level of individual securities. An asset-level momentum strategy ranks all assets on their return in the previous period, buys those assets that did better than average and sells those that did worse. It can be implemented through

$$N_{i,t} = \frac{1}{2n}[\Delta P_{i,t} - \Delta P_{M,t}], \ i = 1, \ldots, 2n, \tag{48}$$

where $N_{i,t}$ is the share position in asset i. An asset-level value strategy buys (sells) those assets trading below (above) fundamental value:

$$N_{i,t} = \frac{1}{2n}[P^*_{i,t} - P_{i,t}], \ \ i = 1, \ldots, 2n. \tag{49}$$

In the presence of switchers, we expect both asset-level momentum and asset-level value strategies to be profitable. If an asset performed well last period, there is good chance that the outperformance was due to the asset's being a member of a "hot" style enjoying inflows from switchers. If so, the style is likely to keep attracting inflows from switchers next period, making it likely that the asset itself also does well next period.

Similarly, an asset trading below fundamental value may be in this predicament because it is a member of a style that is currently unpopular with switchers and is suffering fund outflows. If so, we expect the style and all the assets in it to eventually correct back up to fundamental value, bringing high returns to a value strategy. Specifically, we can prove:

Proposition 6. *The asset-level momentum strategy in* Eq. (48) *and the asset-level value strategy in* Eq. (49) *have strictly positive expected returns in the presence of switchers.*

Table 2 shows that, in our simulated data, these strategies offer attractive Sharpe ratios. The Sharpe ratio we compute is the mean one period change in wealth from implementing the strategy divided by the standard deviation of the one period change in wealth.

Together with our earlier results on comovement, Proposition 6 suggests that our simulated data can match the regression evidence on value stock returns and earnings very closely. Fama and French (1992, 1993, 1995) document a three-part puzzle. First, value stocks earn a premium not captured by the CAPM; second, value stocks comove, and loadings on a specific factor, christened the HML factor, can capture the value premium; and finally, shocks to the common factor in value stock returns are only weakly correlated with shocks to the common factor in value stock earnings news.

Table 2

Sharpe ratios of various stock-picking strategies

Some investors in the economy, known as switchers, group risky assets into two categories, X and Y, known as styles. The table reports Sharpe ratios of certain strategies in this economy, where Sharpe ratio is taken to be the mean one period change in wealth from implementing the strategy divided by the standard deviation of the one period change in wealth. An asset-level momentum strategy buys assets that performed better than average last period; a style-level momentum strategy buys styles that did better than average last period; and a within-style momentum strategy buys assets that did better than their style last period. An asset-level value strategy buys assets trading below fundamental value; a style-level value strategy buys styles trading below fundamental value; and a within-style value strategy buys assets trading at a bigger discount to fundamental value than their style. The optimal strategy is the one that would be chosen by an arbitrageur who knows the correct process followed by prices in the economy.

Strategy	Sharpe ratio
Momentum	
Asset-level	0.61
Style-level	0.61
Within-style	0
Value	
Asset-level	0.12
Style-level	0.12
Within-style	0
Optimal	0.62

To see if our simulated data can replicate this evidence, we run the following three regressions. As usual, we think of X and Y as two fixed styles, such as old economy and new economy stocks:

$$\Delta P_{\text{Val},t} = 0.044 + 1.06\Delta P_{M,t} + u_{\text{Val},t}, \tag{50}$$

$$\Delta P_{\text{Val},t} = -0.009 + 1.00\Delta P_{M,t} + 0.48\Delta P_{S,t} + u_{\text{Val},t},\ R^2 = 92\%, \tag{51}$$

$$\Delta P_{\text{Val},t} = 0.033 + 1.00\varepsilon_{M,t} + 0.48\varepsilon_{S,t} + u_{\text{Val},t},\ R^2 = 50\%. \tag{52}$$

The regression variables are constructed in the following way. Each period, we rank all risky assets on the difference between their prices and fundamental values, $P_{i,t} - P^*_{i,t}$. The 50% of stocks with lower such values, we call value stocks, and split them randomly into two equal-sized groups, $VAL_{A,t}$ and $VAL_{B,t}$. The remaining stocks we call growth stocks, and also split them randomly into $GWT_{A,t}$ and $GWT_{B,t}$. We then define

$$\Delta P_{\text{Val},t+1} = \frac{1}{n/2} \sum_{l_t \in VAL_{A,t}} \Delta P_{l_t,t+1}, \tag{53}$$

$$\Delta P_{S,t+1} = \frac{1}{n/2} \sum_{l_t \in VAL_{B,t}} \Delta P_{l_t,t+1} - \frac{1}{n/2} \sum_{l_t \in GWT_{B,t}} \Delta P_{l,t+1}, \tag{54}$$

$$\varepsilon_{S,t+1} = \frac{1}{n/2} \sum_{l_t \in VAL_{B,t}} \varepsilon_{l_t,t+1} - \frac{1}{n/2} \sum_{l_t \in GWT_{B,t}} \varepsilon_{l_t,t+1}. \tag{55}$$

In words, $\Delta P_{\text{Val},t}$ is the return on a portfolio consisting of half the available universe of n value stocks; $\Delta P_{S,t}$ is a style factor in returns, analogous to the HML factor, constructed as the return on a portfolio of the remaining value stocks minus the return on half the available growth stocks; and $\varepsilon_{S,t}$ is a style factor in cash flows, constructed in a similar way to $\Delta P_{S,t}$. Finally, $\varepsilon_{M,t}$ and $\Delta P_{M,t}$ are as defined in Proposition 1. By splitting value stocks and growth stocks into two random groups, we ensure that $\Delta P_{\text{Val},t}$ and $\Delta P_{S,t}$ are constructed using different stocks and hence that spurious correlation is avoided.

The intercept in Eq. (50) confirms that the value portfolio $\Delta P_{\text{Val},t}$ earns an anomalously high average return, as judged by the CAPM. The positive slope on $\Delta P_{S,t}$ in Eq. (51) shows that there is a common factor in the returns of value stocks, while the tiny intercept shows that loadings on this factor can help capture the value premium. Finally, the drop in R^2 between Eqs. (51) and (52) shows that the common factor in fundamentals lines up poorly with the common factor in returns. Note that (50) and (51) on their own demonstrate that in our model, common factors in returns can be associated with high *average* returns for reasons unrelated to risk.

4.5. Style-level momentum and value strategies

The superior performance of asset-level momentum and value strategies documented in Proposition 6 can also be explained by the feedback models of

De Long et al. (1990a) and Hong and Stein (1999), by the more psychological models of Barberis, Shleifer and Vishny (1998) and Daniel et al. (2001), as well as by the learning model of Lewellen and Shanken (2002). Moreover, the mechanism used in this paper—an excessively negative reaction on the part of some investors to poor prior performance—is similar to that used by De Long et al. (1990a) and Hong and Stein (1999).

In order to make predictions that distinguish our framework, we introduce some additional investment strategies. First, we consider style-level versions of the strategies in Eqs. (48) and (49). A style-level momentum strategy buys into *styles* with good recent performance and avoids styles that have done poorly:

$$N_{i,t} = \frac{1}{2n}\left[\frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2}\right],\ i \in X, \tag{56}$$

$$N_{j,t} = \frac{1}{2n}\left[\frac{\Delta P_{Y,t} - \Delta P_{X,t}}{2}\right],\ j \in Y. \tag{57}$$

A style-level value strategy buys into styles trading below fundamental value and shorts the remaining styles:

$$N_{i,t} = \frac{1}{2n}\left[P^*_{X,t} - P_{X,t}\right],\ i \in X, \tag{58}$$

$$N_{j,t} = \frac{1}{2n}\left[P^*_{Y,t} - P_{Y,t}\right],\ j \in Y. \tag{59}$$

Finally, we also consider "within-style" versions of the strategies in Eqs. (48) and (49). In a within-style momentum strategy, the investor buys those assets which outperformed their style last period and sells those which underperformed their style. It can be implemented through

$$N_{i,t} = \frac{1}{2n}\left[\Delta P_{i,t} - \Delta P_{X,t}\right],\ i \in X, \tag{60}$$

$$N_{j,t} = \frac{1}{2n}\left[\Delta P_{j,t} - \Delta P_{Y,t}\right],\ j \in Y. \tag{61}$$

Correspondingly, a within-style value strategy buys (sells) assets trading at a larger discount (premium) of price to fundamental value than their style:

$$N_{i,t} = \frac{1}{2n}\left(\left[P^*_{i,t} - P_{i,t}\right] - \left[P^*_{X,t} - P_{X,t}\right]\right), i \in X, \tag{62}$$

$$N_{j,t} = \frac{1}{2n}\left(\left[P^*_{j,t} - P_{j,t}\right] - \left[P^*_{Y,t} - P_{Y,t}\right]\right), j \in Y. \tag{63}$$

These new strategies allow us to make predictions that are unique to our framework. In particular, we can prove:

Proposition 7. (i) *Style-level momentum and value strategies offer Sharpe ratios that are greater than or equal to those of their asset-level counterparts*; *and* (ii) *within-style momentum and value strategies are unprofitable*, *offering an expected return of zero.*

Table 2 reports the Sharpe ratios of both style-level and within-style momentum and value strategies, illustrating the proposition for one particular set of parameter values.

The intuition behind Proposition 7 is straightforward. Since mispricing occurs at the level of a style in our model, a strategy designed to exploit this mispricing must be at least as effective when implemented at the style level as it is when implemented at the individual asset level. Moreover, precisely because mispricing is a style-level phenomenon, style-neutral strategies like the within-style strategies will not be able to exploit any mispricing and will remain unprofitable.

Proposition 7 does not hold in traditional positive feedback trading models where the feedback occurs at the individual asset level. In this case, the most effective momentum strategy will buy individual stocks which outperformed last period, in anticipation of further purchases by positive feedback traders. A strategy that simply buys outperforming styles is a less efficient way of picking out future winners and hence offers lower Sharpe ratios. On the other hand, since mispricing occurs at the individual asset level in these models, even style-neutral momentum strategies can be profitable, in contrast to Proposition 7.

In taking Proposition 7 to the data, it is worth keeping in mind that it relies on our simplifying but strong assumption that all noise trading occurs at the style level. In practice, at least some noise trading is likely to be an asset-level phenomenon. In this case, we still expect style-level strategies to do well, although not necessarily better than asset-level strategies, and we expect within-strategies to do less well, although their average return need not be as low as zero.

What evidence there is on style momentum strategies provides some support for style investing. In line with part (i) of Proposition 7, a number of papers find that certain style-level momentum strategies have earned average returns as impressive as those of individual stock-level momentum. Moskowitz and Grinblatt (1999) show that a momentum strategy based on industry portfolios is profitable.[15] Lewellen (2002) investigates momentum strategies using size-sorted and book-to-market sorted portfolios, and claims them both to be at least as profitable as individual-stock momentum. Also consistent with our model, Haugen and Baker (1996) track returns on a large number of investment styles and show that a strategy which tilts towards styles with good prior performance earns high risk-adjusted returns—higher than the returns on any one style. They successfully replicate their findings in out-of-sample tests in a number of international markets.

There is little existing work on style-based value strategies but Asness et al. (1997) show that a value strategy applied to country portfolios works well. Evidence on part (ii) of Proposition 7 is also hard to come by. However, some support comes from

[15] Grundy and Martin (2001) emphasize that the results of Moskowitz and Grinblatt (1999) depend heavily on the positive autocorrelation of monthly industry returns at the first lag. If a gap of a month is inserted between the portfolio formation period and the portfolio test period, individual stock-level momentum is profitable, while industry-level momentum is less so. This suggests that momentum in individual stock returns cannot be purely a style effect.

Moskowitz and Grinblatt (1999), who find that within-industry momentum strategies are unprofitable.

It is also possible to relate the style momentum and style value strategies to the optimal strategy followed by an arbitrageur clever enough to figure out the correct process for asset prices in our economy.

Proposition 8. *The optimal strategy for an arbitrageur who knows that prices follow* Eqs. (24) and (26) *is given by the following share demands*:

$$N^{\mathrm{A}}_{i,t} = \frac{c}{n}\left[\frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2} - (1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}\right)\right], i \in X, \tag{64}$$

$$N^{\mathrm{A}}_{j,t} = \frac{c}{n}\left[\frac{\Delta P_{Y,t} - \Delta P_{X,t}}{2} - (1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{Y,t-k} - \Delta P_{X,t-k}}{2}\right)\right], j \in Y, \tag{65}$$

where c is a positive constant.[16]

If we use $N^{\mathrm{mom}}_{i,t}$ and $N^{\mathrm{val}}_{i,t}$ to denote the share demands of the style-level momentum and value strategies in Eqs. (56) and (58), respectively, then Eq. (64) together with Eqs. (24) and (26) imply that

$$N^{\mathrm{A}}_{i,t} = 2c\left[N^{\mathrm{mom}}_{i,t} + \phi(1-\theta)N^{\mathrm{val}}_{i,t}\right], \forall i. \tag{66}$$

In words, the optimal strategy is a constant combination of style momentum and style value.

Eq. (66) shows that when θ is close to 1, as in our parameterized example, the optimal strategy is very similar to style-level momentum. This makes intuitive sense. When switcher flows are very persistent and mispricings take a long time to correct, it makes more sense for an arbitrageur to ride with the switchers than to bet against them. It is also consistent with Table 2, which shows that the style momentum and optimal strategies have very similar Sharpe ratios. For lower values of θ, switcher flows are less persistent and prices revert to fundamental value more quickly. This suggests that for low θ, the value strategy should be relatively more attractive. Eq. (66) confirms that as θ falls, value becomes more attractive relative to momentum in the sense that the optimal strategy places relatively more weight on value.[17]

5. Special styles

An important style that deserves further discussion in indexation. In Section 4, we informally applied some of our propositions to the case where one of the styles is an

[16] The "A" superscript in these expressions stands for arbitrageur.

[17] This observation requires some computation, because ϕ is itself a function of θ. We find that ϕ, and therefore $\phi(1-\theta)$, is a decreasing function of θ.

index, but it may be helpful to restate our predictions about this style more explicitly. Setting $X = I$ and $Y = NI$, where I and NI represent assets within a certain index and assets outside that index, respectively, gives:

Proposition 9. (i) *Suppose that asset* j, *not previously a member of an index* I, *is reclassified as belonging to* I. *Then* $\text{cov}(\Delta P_{j,t},\ \Delta P_{I,t})$ *increases after* j *is added to* I; (ii) *If asset* i *is in an index* I *while asset* j *is not, then*

$$\text{corr}(\Delta P_{i,t} - \Delta P_{M,t},\ \Delta P_{j,t} - \Delta P_{M,t}) < \text{corr}(\varepsilon_{i,t} - \varepsilon_{M,t},\ \varepsilon_{j,t} - \varepsilon_{M,t}); \tag{67}$$

(iii) *For some* $K \geqslant 1$,

$$\begin{aligned}
&\text{corr}(\Delta P_{I,t}, \Delta P_{I,t-k}) > 0,\ 1 \leqslant k \leqslant K,\\
&\text{corr}(\Delta P_{I,t}, \Delta P_{I,t-k}) < 0,\ k = K+1,\\
&\text{corr}(\Delta P_{I,t}, \Delta P_{NI,t-k}) < 0,\ 1 \leqslant k \leqslant K,\\
&\text{corr}(\Delta P_{I,t}, \Delta P_{NI,t-k}) > 0,\ k = K+1.
\end{aligned} \tag{68}$$

In words, a stock which is added to an index should comove more with the index after inclusion than before; the returns on a stock in an index should comove with the returns on a stock not in the index less than their fundamentals do; indices should have positive (negative) own-autocorrelations at short (long) horizons; and they should be negatively (positively) cross-autocorrelated with stocks outside the index at short (long) horizons. Moreover, since the importance of indexing has grown over time, these phenomena should be stronger in more recent data samples. Vijh (1994) and Barberis et al. (2001) examine some of these predictions empirically using data for the S&P 500 index and find supportive evidence.

Another interesting issue arises with *price-dependent* styles, where the characteristic defining the style depends on price. Many common styles such as "small stocks" or "value stocks" fall into this category. When a style is price-dependent, its composition changes. Suppose that switchers have kicked off a long upswing in the price of small stocks relative to their fundamentals. They buy small stocks, pushing up their price, which attracts more switchers, and so on. After a while, some of the small stocks experience price increases so large that they cannot be considered small any more and are no longer part of the small stock style.

This change in composition need not brake the evolution of the small stock style itself. However, it may mean that the degree of misvaluation experienced by any individual asset is *lower* than in the case where the style characteristic is not price-dependent. If a small stock becomes too highly valued relative to its fundamentals, it ceases to be a small stock and the buying pressure from switchers following the small stock style eases off, halting its ascent. This argument depends on the correlation between characteristic and price being negative, so that the higher a stock's price, the less likely it is to be a small stock. When this correlation is positive, misvaluation of individual stocks is more severe when the style is price-dependent than when it is not.

6. Conclusion

The model of financial markets discussed in this paper is in many ways similar to that proposed by Black (1986). On the one hand, financial markets in our economy are not efficient. Prices deviate substantially from fundamental values as styles become popular or unpopular. For an arbitrageur with a good model of prices, there are substantial profits to be made from a combination of contrarian and momentum trading. On the other hand, despite the fact that markets are inefficient, prices are very noisy. Patterns in security prices are complex and change significantly over time. Without knowing which style or model is favored, arbitrage is risky and consistent profits hard to come by. To some people, such markets might even appear efficient.

In this paper, we have tried to show that these markets are not entirely anarchic. They do exhibit long run pressures toward fundamentals and there are empirical predictions one can make about them, such as excess comovement within styles and non-trivial autocorrelation patterns in style returns. Further exploration of style investing is likely to generate many more predictions, and may offer new ways of looking at existing empirical facts.

Appendix A

Proof of Proposition 1. From Eq. (24), for any $i \in X$,

$$\Delta P_{i,t+1} = \varepsilon_{i,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.1}$$

where

$$\Delta N^S_{X,t+1} = \frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2} - (1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}\right) \tag{A.2}$$

is known at time t. Since

$$\Delta P_{M,t+1} = \varepsilon_{M,t+1} = \frac{1}{2n}\sum_{l=1}^{2n}\varepsilon_{l,t+1}, \tag{A.3}$$

we have, for any distinct $i, j \in X$,

$$\Delta P_{i,t+1} - \Delta P_{M,t+1} = \varepsilon_{i,t+1} - \varepsilon_{M,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.4}$$

$$\Delta P_{j,t+1} - \Delta P_{M,t+1} = \varepsilon_{j,t+1} - \varepsilon_{M,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}. \tag{A.5}$$

This implies

$$\begin{aligned}\mathrm{cov}(\Delta P_{i,t+1} - \Delta P_{M,t+1},\ \Delta P_{j,t+1} - \Delta P_{M,t+1}) &= \mathrm{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) \\ &\quad + \frac{1}{\phi^2}\mathrm{var}(\Delta N^S_{X,t+1}),\end{aligned} \tag{A.6}$$

$$\mathrm{var}(\Delta P_{l,t+1} - \Delta P_{M,t+1}) = \mathrm{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) + \frac{1}{\phi^2}\mathrm{var}(\Delta N^S_{X,t+1}), \quad l \in \{i, j\}. \tag{A.7}$$

The proposition therefore follows if

$$\mathrm{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) < \mathrm{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}). \tag{A.8}$$

Using

$$\varepsilon_{i,t+1} - \varepsilon_{M,t+1} = \frac{\psi_S f_{X,t+1} - \psi_S f_{Y,t+1}}{2} + \sqrt{1 - \psi_M^2 - \psi_S^2}\left(f_{i,t+1} - \frac{1}{2n}\sum_{l=1}^{2n} f_{l,t+1}\right), \tag{A.9}$$

it is easily checked that

$$\mathrm{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) = \frac{\psi_S^2}{2} - \frac{1 - \psi_M^2 - \psi_S^2}{2n}, \tag{A.10}$$

$$\mathrm{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) = \frac{\psi_S^2}{2} + \frac{2n-1}{2n}(1 - \psi_M^2 - \psi_S^2), \tag{A.11}$$

which means that inequality (A.8) does indeed hold.

Proof of Proposition 2. Suppose that asset $n+1$ is reclassified from style Y into style X, and that at the same time, asset 1 is reclassified from style X into style Y. Before reclassification, we have

$$\Delta P_{X,t+1} = \varepsilon_{X,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.12}$$

$$\Delta P_{n+1,t+1} = \varepsilon_{n+1,t+1} - \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.13}$$

where

$$\varepsilon_{X,t+1} = \frac{1}{n}\sum_{l \in X} \varepsilon_{l,t+1}. \tag{A.14}$$

This implies that before reclassification,

$$\mathrm{cov}(\Delta P_{n+1,t},\ \Delta P_{X,t}) = \psi_M^2 - \frac{1}{\phi^2}\mathrm{var}\left(\Delta N^S_{X,t}\right). \tag{A.15}$$

After reclassification, we have

$$\Delta P_{X,t+1} = \varepsilon_{X,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.16}$$

$$\Delta P_{n+1,t+1} = \varepsilon_{n+1,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.17}$$

which implies

$$\text{cov}(\Delta P_{n+1,t},\ \Delta P_{X,t}) = \psi_M^2 + \frac{1-\psi_M^2}{n} + \frac{1}{\phi^2}\text{var}(\Delta N^S_{X,t+1}). \tag{A.18}$$

Therefore, $\text{cov}(\Delta P_{n+1,t},\ \Delta P_{X,,t})$ does indeed increase after addition.

Proof of Proposition 3. For $i \in X$ and $j \in Y$,

$$\Delta P_{i,t+1} - \Delta P_{M,t+1} = \varepsilon_{i,t+1} - \varepsilon_{M,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{A.19}$$

$$\Delta P_{j,t+1} - \Delta P_{M,t+1} = \varepsilon_{j,t+1} - \varepsilon_{M,t+1} - \frac{\Delta N^S_{X,t+1}}{\phi}. \tag{A.20}$$

This implies

$$\begin{aligned}\text{cov}(\Delta P_{i,t+1} - \Delta P_{M,t+1}\ \Delta P_{j,t+1} - \Delta P_{M,t+1}) = {} & \text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) \\ & - \frac{1}{\phi^2}\text{var}\left(\Delta N^S_{X,t+1}\right)\end{aligned} \tag{A.21}$$

$$\begin{aligned}\text{var}(\Delta P_{l,t+1} - \Delta P_{M,t+1}) = {} & \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) \\ & + \frac{1}{\phi^2}\text{var}\left(\Delta N^S_{X,t+1}\right), \quad l \in \{i,j\}.\end{aligned} \tag{A.22}$$

The proposition therefore follows if

$$-\text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) < \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}). \tag{A.23}$$

Using Eq. (A.9) and

$$\begin{aligned}\varepsilon_{j,t+1} - \varepsilon_{M,t+1} = {} & \frac{\psi_S f_{Y,t+1} - \psi_S f_{X.t+1}}{2} \\ & + \sqrt{1-\psi_M^2-\psi_S^2}\left(f_{j,t+1} - \frac{1}{2n}\sum_{l=1}^{2n} f_{l,t+1}\right),\end{aligned} \tag{A.24}$$

it is easily checked that

$$\text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1},\ \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) = -\left(\frac{\psi_S^2}{2} + \frac{1-\psi_M^2-\psi_S^2}{2n}\right), \tag{A.25}$$

$$\text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) = \frac{\psi_S^2}{2} + \frac{2n-1}{2n}(1-\psi_M^2-\psi_S^2), \tag{A.26}$$

which means that inequality (A.23) does indeed hold.

We now prove the following lemma, which will be useful in the remaining proofs.

Lemma. *In any stationary equilibrium with* $\theta > 0$, *it must be true that* $0 < \theta < 1$ *and* $\phi > 1$.

Proof of Lemma. It is easily checked using Eqs. (24) and (26) that $\rho_1 > \rho_2$, where ρ_1 and ρ_2 are defined in Eq. (20). Eq. (25) then immediately implies that $\phi > 0$ in any stationary equilibrium.

From Eq. (24), we can write

$$\Delta P_{X,t+1} - \Delta P_{Y,t+1} = (\varepsilon_{X,t+1} - \varepsilon_{Y,t+1}) + \frac{\Delta P_{X,t} - \Delta P_{Y,t}}{\phi} - \frac{(1-\theta)}{\phi}\sum_{k=1}^{t-1}\theta^{k-1}(\Delta P_{X,t-k} - \Delta P_{Y,t-k}), \tag{A.27}$$

which then implies

$$\Delta P_{X,t+1} - \Delta P_{Y,t+1} = \left(\theta + \frac{1}{\phi}\right)(\Delta P_{X,t} - \Delta P_{Y,t}) - \frac{1}{\phi}(\Delta P_{X,t-1} - \Delta P_{Y,t-1}) + (\varepsilon_{X,t+1} - \varepsilon_{Y,t+1}) - \theta(\varepsilon_{X,t} - \varepsilon_{Y,t}). \tag{A.28}$$

Using standard theory (see Hamilton, 1994), $\Delta P_{X,t} - \Delta P_{Y,t}$ will be a stable process so long as the roots of

$$\lambda^2 - \lambda\left(\theta + \frac{1}{\phi}\right) + \frac{1}{\phi} = 0 \tag{A.29}$$

are all less than one in absolute magnitude. Within the range $\theta > 0$, $\phi > 0$, this will be true so long as

$$0 < \theta < 1, \ \phi > 1. \tag{A.30}$$

Proof of Propositions 4 and 5. We use the notation

$$\hat{\Gamma}_k = \mathrm{cov}(\Delta P_{X,t} - \Delta P_{Y,t}, \ \Delta P_{X,t+k} - \Delta P_{Y,t+k}), \ k \geqslant 0, \tag{A.31}$$

$$\Gamma_k = \mathrm{cov}(\Delta P_{X,t}, \ \Delta P_{X,t+k}), \ k \geqslant 0, \tag{A.32}$$

$$\gamma_k = \mathrm{corr}(\Delta P_{X,t}, \ \Delta P_{X,t+k}), \ k \geqslant 0. \tag{A.33}$$

Note that since

$$\mathrm{cov}\left(\Delta P_{X,t}, \ \Delta P_{X,t+k}\right) = -\mathrm{cov}(\Delta P_{X,t}, \ \Delta P_{Y,t+k}), \ k \geqslant 1, \tag{A.34}$$

it follows that

$$\hat{\Gamma}_k = 4\Gamma_k, \ \geqslant 1. \tag{A.35}$$

To prove the first part of the proposition, it suffices to show that $\gamma_1 > 0$ and that $\gamma_{K+1} < 0$ for some $K \geqslant 1$. Part (ii) of the proposition then follows immediately from Eq. (A.34).

First, we show that $\gamma_1 > 0$. Computing the covariance of Eq. (A.28) with $\Delta P_{X,t+1} - \Delta P_{Y,t+1}$, $\Delta P_{X,t} - \Delta P_{Y,t}$ and $\Delta P_{X,t-1} - \Delta P_{Y,t-1}$ in turn, gives

$$\hat{\Gamma}_0 = \left(\theta + \frac{1}{\phi}\right)\hat{\Gamma}_1 - \frac{1}{\phi}\hat{\Gamma}_2 + \left(1 - \frac{\theta}{\phi}\right)(2\psi_s^2 + k_0), \tag{A.36}$$

$$\hat{\Gamma}_1\left(1+\frac{1}{\phi}\right) = \left(\theta+\frac{1}{\phi}\right)\hat{\Gamma}_0 - \theta(2\psi_s^2 + k_0), \quad \text{(A.37)}$$

$$\hat{\Gamma}_2 = \left(\theta+\frac{1}{\phi}\right)\hat{\Gamma}_1 - \frac{1}{\phi}\hat{\Gamma}_0, \quad \text{(A.38)}$$

where

$$k_0 = \frac{2}{n}(1-\psi_S^2-\psi_M^2). \quad \text{(A.39)}$$

This gives us three equations in three unknowns, and after some algebra, we obtain

$$\hat{\Gamma}_1 = \frac{(2\psi_S^2+k_0)(1+\theta)}{(\phi-1)(1+\theta+2/\phi)}, \quad \text{(A.40)}$$

which is positive under the restrictions $0<\theta<1$ and $\phi>1$ that we derived in the preceding lemma. Therefore, $\gamma_1>0$.

To show that $\gamma_{K+1}<0$ for some $K\geqslant 1$, it is sufficient to show that

$$\pi = \hat{\Gamma}_2 + \theta\hat{\Gamma}_3 + \theta^2\hat{\Gamma}_4 + \cdots < 0. \quad \text{(A.41)}$$

Taking the covariance of Eq. (A.27) with $\Delta P_{X,t+1} - \Delta P_{Y,t+1}$, we obtain

$$\hat{\Gamma}_0 = 2\psi_S^2 + k_0 + \frac{\hat{\Gamma}_1}{\phi} - \frac{1-\theta}{\phi}\pi. \quad \text{(A.42)}$$

Substituting in $\hat{\Gamma}_1$ from Eq. (A.40) and the implied reduced form for $\hat{\Gamma}_0$ from (A. 37) gives

$$\begin{aligned}\pi &= -\left(\frac{\phi}{1-\theta}\right)\left(\hat{\Gamma}_0 - (2\psi_S^2+k_0) - \frac{\hat{\Gamma}_1}{\phi}\right)\\ &= \frac{-(2\psi_S^2+k_0)}{(\phi-1)(1+\theta+2/\phi)},\end{aligned} \quad \text{(A.43)}$$

which is indeed negative under the restrictions $0<\theta<1$ and $\phi>1$ derived in the lemma. This concludes the proof of Proposition 4. The proof of Proposition 5 is identical in structure.

Proof of Proposition 6. Define

$$\Lambda_k = \mathrm{cov}(\Delta P_{i,t},\ \Delta P_{i,t+k}). \quad \text{(A.44)}$$

Then using Eq. (A.1), it is simple to show that for $k\geqslant 1$,

$$\mathrm{cov}(\Delta P_{i,t},\ \Delta P_{j,t+k}) = \Lambda_k \text{ for all } i,\ j \text{ in the same style}, \quad \text{(A.45)}$$

$$\mathrm{cov}(\Delta P_{i,t},\ \Delta P_{j,t+k}) = -\Lambda_k \text{ for all } i,\ j \text{ in different styles} \quad \text{(A.46)}$$

and

$$\Lambda_k = \Gamma_k. \quad \text{(A.47)}$$

The expected return of the asset-level momentum strategy is given by

$$\begin{aligned}
\mathrm{E}\left(\sum_{i=1}^{2n} N_{i,t}\Delta P_{i,t+1}\right) &= \frac{1}{2n}\mathrm{E}\sum_{i=1}^{2n}(\Delta P_{i,t}-\Delta P_{M,t})\Delta P_{i,t+1}\\
&= \frac{1}{2n}\sum_{i=1}^{2n}\mathrm{E}(\Delta P_{i,t}\Delta P_{i,t+1}) - \frac{1}{4n^2}\sum_{i,j=1}^{2n}\mathrm{E}(\Delta P_{i,t}\Delta P_{j,t+1})\\
&= \frac{1}{2n}\sum_{i=1}^{2n}\mathrm{cov}(\Delta P_{i,t},\Delta P_{i,t+1}) - \frac{1}{4n^2}\sum_{i,j=1}^{2n}\mathrm{cov}(\Delta P_{i,t},\Delta P_{j,t+1})\\
&\quad + \frac{1}{2n}\sum_{i=1}^{2n}(\mu_i-\mu_M)^2\\
&= \Lambda_1 + 0 + \frac{1}{2n}\sum_{i=1}^{2n}(\mu_i-\mu_M)^2 = \Gamma_1,
\end{aligned} \tag{A.48}$$

where μ_i is the mean return of asset i and μ_M is the mean return of the market portfolio of all risky assets. The last equality follows because in our simple economy, all assets have the same mean return μ_i. In the proof of Proposition 4, we showed that $\Gamma_1 > 0$, which means that the expected return is indeed positive.

The expected return of the asset-level value strategy is given by

$$\begin{aligned}
\mathrm{E}\left(\sum_{i=1}^{2n} N_{i,t}\Delta P_{i,t+1}\right) &= \frac{1}{2n}\mathrm{E}\left(\sum_{i=1}^{n}\left(-\frac{1}{\phi}\sum_{k=1}^{t-1}\theta^{k-1}\frac{\Delta P_{X,t-k}-\Delta P_{Y,t-k}}{2}\right)\Delta P_{i,t+1}\right)\\
&\quad - \frac{1}{2n}\mathrm{E}\left(\sum_{i=n+1}^{2n}\left(-\frac{1}{\phi}\sum_{k=1}^{t-1}\theta^{k-1}\frac{\Delta P_{X,t-k}-\Delta P_{Y,t-k}}{2}\right)\Delta P_{i,t+1}\right)\\
&= \frac{1}{2}\mathrm{E}\left[\left(-\frac{1}{\phi}\sum_{k=1}^{t-1}\theta^{k-1}\frac{\Delta P_{X,t-k}-\Delta P_{Y,t-k}}{2}\right)(\Delta P_{X,t+1}-\Delta P_{Y,t+1})\right] \qquad \text{(A.49)}\\
&= -\frac{1}{4\phi}(\hat{\Gamma}_2+\theta\hat{\Gamma}_3+\theta^2\hat{\Gamma}_4+\cdots) - \frac{1-\theta}{4\phi}(\mu_X-\mu_Y)^2\\
&= -\frac{\pi}{4\phi},
\end{aligned}$$

where μ_X and μ_Y are the mean returns of styles X and Y, respectively. The last equality follows because all securities have the same expected return in our economy. In the proof of Proposition 4, we showed that $\pi < 0$, which implies that the expected return of the asset-level value strategy is indeed positive.

Proof of Proposition 7 (Part (i)). This part of the proposition is trivially true for value strategies since the share demands of a style-level value strategy are identical to the share demands of an asset-level value strategy.

We now show that the Sharpe ratio of a style-level momentum strategy is strictly greater than that of the asset-level momentum strategy. We do this by showing that both strategies have the same expected return, but that the style-level strategy has a lower expected *squared* return, and hence a lower variance.

The expected return of a style-level momentum strategy is

$$\begin{aligned}
&\mathrm{E}\left(\sum_{i=1}^{n}\frac{1}{2n}\left(\frac{\Delta P_{X,t}-\Delta P_{Y,t}}{2}\right)\Delta P_{i,t+1}+\sum_{i=n+1}^{2n}\frac{1}{2n}\left(\frac{\Delta P_{Y,t}-\Delta P_{X,t}}{2}\right)\Delta P_{i,t+1}\right)\\
&\quad=\frac{1}{4}\bar{\mathrm{E}}[(\Delta P_{X,t}-\Delta P_{Y,t})(\Delta P_{X,t+1}-\Delta P_{Y,t+1})]\\
&\quad=\Gamma_1+\frac{1}{4}(\mu_X-\mu_Y)^2=\Gamma_1.
\end{aligned}\tag{A.50}$$

This is indeed equal to the expected return of the asset-level momentum strategy, computed in the proof of Proposition 6.

The expected *squared* return of a style-level momentum strategy is given by

$$\begin{aligned}
&\frac{1}{16}\mathrm{E}[(\Delta P_{X,t}-\Delta P_{Y,t})^2(\Delta P_{X,t+1}-\Delta P_{Y,t+1})^2]\\
&=\mathrm{E}[(\Delta P_{X,t}-\Delta P_{M,t})^2(\Delta P_{X,t+1}-\Delta P_{M,t+1})^2]\\
&=\mathrm{cov}[(\Delta P_{X,t}-\Delta P_{M,t})^2,\ (\Delta P_{X,t+1}-\Delta P_{M,t+1})^2]\\
&\quad+\mathrm{E}(\Delta P_{X,t}-\Delta P_{M,t})^2\mathrm{E}(\Delta P_{X,t+1}-\Delta P_{M,t+1})^2.
\end{aligned}\tag{A.51}$$

Substituting in

$$\Delta P_{X,t+1}-\Delta P_{M,t+1}=\varepsilon_{X,t+1}-\varepsilon_{M,t+1}+\frac{\Delta N^S_{X,t+1}}{\phi}\tag{A.52}$$

and taking the expectation, expression (A.51) eventually reduces to

$$\begin{aligned}
&\mathrm{cov}\left(\frac{\Delta N^2_{X,t+1}}{\phi^2},(\varepsilon_{X,t}-\varepsilon_{M,t})^2+2(\varepsilon_{X,t}-\varepsilon_{M,t})\frac{\Delta N_{X,t}}{\phi}+\frac{\Delta N^S_{X,t}}{\phi^2}\right)\\
&\quad+\left(\frac{\psi_S^2}{2}+\frac{1}{2n}(1-\psi_M^2-\psi_S^2)+\frac{\mathrm{var}(\Delta N_{X,t})}{\phi^2}\right)^2.
\end{aligned}\tag{A.53}$$

The expected squared return of the asset-level momentum strategy is given by

$$\begin{aligned}
&\frac{1}{4n^2}\mathrm{E}\left(\sum_{i,j=1}^{2n}(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t})\Delta P_{i,t+1}\Delta P_{j,t+1}\right)\\
&=\frac{1}{4n^2}\sum_{i,j=1}^{2n}\mathrm{cov}[(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t}),\Delta P_{i,t+1}\Delta P_{j,t+1}]\\
&\quad+\frac{1}{4n^2}\sum_{i,j=1}^{2n}\mathrm{E}[(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t})]\mathrm{E}[\Delta P_{i,t+1}\Delta P_{j,t+1}].
\end{aligned}\tag{A.54}$$

Substituting in the expressions for $\Delta P_{i,t} - \Delta P_{M,t}$ and $\Delta P_{i,t+1}$ from Eqs. (A.1) and (A.20), the expected squared return reduces to

$$\text{cov}\left(\frac{\Delta N_{X,t+1}^2}{\phi^2}, (\varepsilon_{X,t} - \varepsilon_{M,t})^2 + 2(\varepsilon_{X,t} - \varepsilon_{M,t})\frac{\Delta N_{X,t}}{\phi} + \frac{\Delta N_{X,t}^2}{\phi^2}\right)$$
$$+ \left(\frac{\psi_S^2}{2} + \frac{\text{var}(\Delta N_{X,t})}{\phi^2}\right)$$
$$+ \frac{1 - \psi_M^2 - \psi_S^2}{4n^2}\left(\frac{4n\,\text{var}(\Delta N_{X,t})}{\phi^2} + (2n-1)(1 - \psi_M^2 - \psi_S^2) + 2n\psi_S^2\right). \quad \text{(A.55)}$$

It is now simple to show that expression (A.55) is strictly greater than expression (A.53). The style-level momentum strategy does indeed have the lower variance and hence the higher Sharpe ratio.

Proof of Proposition 7 (Part (ii)). This part of the proposition is trivially true for value strategies since the share demands of a within-style value strategy are identically zero. The expected return of the within-style momentum strategy is

$$\frac{1}{2n}\text{E}\left(\sum_{i=1}^{n}(\Delta P_{i,t} - \Delta P_{X,t})\Delta P_{i,t+1} + \sum_{i=n+1}^{2n}(\Delta P_{i,t} - \Delta P_{Y,t})\Delta P_{i,t+1}\right)$$
$$= \sum_{i=1}^{n}(\mu_i - \mu_X)^2 + \sum_{i=n+1}^{2n}(\mu_i - \mu_Y)^2 = 0 \quad \text{(A.56)}$$

in our economy, since all stocks have the same expected return.

Proof of Proposition 8. Given wealth of W_t at time t, the arbitrageur solves

$$\max_{N_{i,t}} \text{E}_t^{\text{A}}\left(-\exp\left[-\gamma\left(W_t + \sum_i N_{i,t}\Delta P_{i,t+1}\right)\right]\right) \quad \text{(A.57)}$$

to obtain

$$N_t^{\text{A}} = \frac{(V_t^A)^{-1}}{\gamma}\text{E}_t^{\text{A}}(\Delta P_{t+1}) \quad \text{(A.58)}$$

where N_t^{A} is the vector of optimal demands, V_t^{A} is the arbitrageur's estimate of the conditional covariance matrix of price changes, and where the "A" superscript in these expressions stands for arbitrageur.

Since he knows that prices are determined by Eq. (24), he is able to conclude that

$$E_t^{\text{A}}(\Delta P_{t+1}) = \left(\frac{\Delta N_{X,t+1}^S}{\phi}, \ldots, \frac{\Delta N_{X,t+1}^S}{\phi}, -\frac{\Delta N_{X,t+1}^S}{\phi}, \ldots, -\frac{\Delta N_{X,t+1}^S}{\phi}\right), \quad \text{(A.59)}$$

$$V_t^A = \Sigma_D. \quad \text{(A.60)}$$

Eq. (A.58) then reduces to

$$N_{i,t}^{\text{A}} = \frac{c}{n}\Delta N_{X,t+1}^S, \; i \in X, \quad \text{(A.61)}$$

$$N^{A}_{j,t} = -\frac{c}{n}\Delta N^{S}_{X,t+1},\ j \in Y, \tag{A.62}$$

where c is a positive constant that depends on ψ^2_M, ψ^2_S, γ, and ϕ.

Proof of Proposition 9. This proposition follows directly from Propositions 2–4, with X taken to be the index I, and Y taken to be the set of stocks outside the index, NI.

References

Asness, C., Liew, J., Stevens, R., 1997. Parallels between the cross-sectional predictability of stock returns and country returns. Journal of Portfolio Management 23, 79–87.

Banz, R., 1979. The relationship between return and market value of common stocks. Journal of Financial Economics 9, 3–18.

Barberis, N., Shleifer, A., 2000. Style investing. NBER working paper no. 8039.

Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment. Journal of Financial Economics 49, 307–343.

Barberis, N., Shleifer, A., Wurgler, J., 2001. Comovement. Unpublished working paper. University of Chicago.

Bernstein, R., 1995. Style Investing. Wiley, New York.

Black, F., 1986. Noise. Journal of Finance 41, 529–43.

Brennan, M., Xia, Y., 2001. Stock price volatility and the equity premium. Journal of Monetary Economics 47, 249–283.

Brown, S., Goetzmann, W., 1997. Mutual fund styles. Journal of Financial Economics 43, 373–399.

Brown, S., Goetzmann, W., 2001. Hedge funds with styles. Unpublished working paper. Yale University.

Campbell, J.Y., Lettau, M., Malkiel, B., Xu, Y., 2001. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. Journal of Finance 56, 1–43.

Chan, L., Chen, H., Lakonishok, L., 2002. On mutual fund investment styles. Review of Financial Studies 15, 1407–1437.

Chan, L., Karceski, J., Lakonishok, J., 2000. New paradigm or same old hype in equity investing? Financial Analysis Journal 56, 23–36.

Choe, H., Kho, B.C., Stulz, R., 1999. Do foreign investors destabilize stock markets? Journal of Financial Economics 54, 227–264.

Cutler, D., Poterba, J., Summers, L., 1991. Speculative dynamics. Review of Economic Studies 58, 529–546.

Daniel, K., Hirshleifer, D., Subrahmanyam, A., 2001. Overconfidence, arbitrage, and equilibrium asset pricing. Journal of Finance 53, 1839–1885.

Daniel, K., Titman, S., 1997. Evidence on the characteristics of cross sectional variation in stock returns. Journal of Finance 52, 1–33.

De Long, J.B., Shleifer, A., Summers, L., Waldmann, R., 1990a. Positive feedback investment strategies and destabilising rational speculation. Journal of Finance 45, 375–395.

De Long, J.B., Shleifer, A., Summers, L., Waldmann, R., 1990b. Noise trader risk in financial markets. Journal of Political Economy 98, 703–738.

Fama, E., French, K., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427–465.

Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E., French, K., 1995. Size and book-to-market factors in earnings and returns. Journal of Finance 50, 131–155.

Froot, K., Dabora, E., 1999. How are stock prices affected by the location of trade? Journal of Financial Economics 53, 189–216.

Froot, K., O'Connell, P., Seasholes, M., 2001. The portfolio flows of international investors. Journal of Financial Economics 59, 149–193.
Gompers, P., Metrick, A., 2001. Institutional investors and equity prices. Quarterly Journal of Economics 116, 229–259.
Grundy, B., Martin, J.S., 2001. Understanding the nature of the risks and the sources of the rewards to momentum investing. Review of Financial Studies 14, 29–78.
Hamilton, J., 1994. Time Series Analysis. Princeton University Press, New Jersey.
Harris, L., Gurel, E., 1986. Price and volume effects associated with changes in the S&P 500: new evidence for the existence of price pressure. Journal of Finance 41, 851–860.
Haugen, R., Baker, N., 1996. Commonality in the determinants of expected stock returns. Journal of Financial Economics 41, 401–439.
Hong, H., Stein, J., 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. Journal of Finance 54, 2143–2184.
Kaul, A., Mehrotra, V., Morck, R., 2000. Demand curves for stocks do slope down: new evidence from an index weights adjustment. Journal of Finance 55, 893–912.
Kyle, A., Xiong, W., 2001. Contagion as a wealth effect. Journal of Finance 56, 1401–1443.
Lamont, O., Thaler, R., 2003. Can the market add and subtract? Mispricing in tech stock carveouts. Journal of Political Economy, forthcoming.
Lee, C., Shleifer, A., Thaler, R., 1991. Investor sentiment and the closed-end fund puzzle. Journal of Finance 46, 75–110.
Lewellen, J., 2002. Momentum and autocorrelation in stock returns. Review of Financial Studies 15, 533–563.
Lewellen, J., Shanken, J., 2002. Learning, asset-pricing tests, and market efficiency. Journal of Finance 57, 1113–1145.
Lo, A., MacKinlay, A.C., 1988. Stock market prices do not follow random walks: evidence from a simple specification test. Review of Financial Studies 1, 41–66.
Moskowitz, T., Grinblatt, M., 1999. Do industries explain momentum? Journal of Finance 54, 1249–1290.
Mullainathan, S., 2000. Thinking through categories. Unpublished working paper. MIT.
Peng, L., Xiong, W., 2002. Capacity-constrained learning and asset price comovement. Unpublished working paper. Princeton University.
Pindyck, R., Rotemberg, J., 1990. The excess comovement of commodity prices. Economic Journal 100, 1173–1189.
Poterba, J., Summers, L., 1988. Mean reversion in stock returns: evidence and implications. Journal of Financial Economics 22, 27–59.
Rosch, E., Lloyd, B., 1978. Cognition and Categorization. Lawrence Erlbaum Associates, New Jersey.
Sharpe, W., 1992. Asset allocation: management style and performance measurement. Journal of Portfolio Management 18, 7–19.
Shiller, R., 1989. Comovements in stock prices and comovements in dividends. Journal of Finance 44, 719–729.
Shleifer, A., 1986. Do demand curves for stocks slope down? Journal of Finance 41, 579–590.
Shleifer, A., Vishny, R., 1997. The limits of arbitrage. Journal of Finance 52, 35–55.
Siegel, J., 1999. Stocks for the Long Run. McGraw Hill, New York.
Swensen, D., 2000. Pioneering Portfolio Management. The Free Press, New York.
Tversky, A., Kahneman, D., 1974. Judgement under uncertainty: heuristics and biases. Science 185, 1124–1131.
Veronesi, P., 1999. Stock market overreaction to bad news in good times: a rational expectations equilibrium model. Review of Financial Studies 12, 975–1007.
Vijh, A., 1994. S&P trading strategies and stock betas. Review of Financial Studies 7, 215–251.
Wilson, R., Keil, F., 1999. The MIT Encyclopedia of the Cognitive Sciences. MIT Press, Cambridge. MA.